Properties of Determinants

P-1: The value of the determinant remains unchanged if its row and columns are interchanged.

$C_i \leftrightarrow R_i$

P-2: If any two rows (or columns) of a determinant are interchanged, then sign of determinant changes.

Denote interchange of rows by: $R_i \leftrightarrow R_j$

P-3: If any two rows (or columns) of a determinant are identical (all corresponding elements are same), then value of determinant is zero

P-4: If each element of a row (or a column) of a determinant is multiplied by a constant k, then its value gets multiplied by k.

P-5: If some or all elements of a row or column of a determinant are expressed as sum of two (or more) terms, then the determinant can be expressed as sum of two (or more) determinants.

P-6: If, to each element of any row or column of a determinant, the equimultiples of corresponding elements of other row (or column) are added, then value of determinant remains the same, i.e. the value of determinant remain same if we apply the operation $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$

Remarks:

i) If Δ_1 is the determinant obtained by applying $R_i \to kR_i$ or $C_i \to kC_i$ to the determinant Δ, then $\Delta_1 = k\Delta$

ii) If more than one operation like $R_i \to R_i + kR_j$ is done in one step, care should be taken to see that a row that is affected in one operation should not be used in another operation. A similar remark applies to column operations.

Adjoint of a square matrix $A = [a_{ij}]_{n \times n}$ is defined as the transpose of the matrix $[A_{ij}]_{n \times n}$, where A_{ij} is the cofactor of the element a_{ij}.

↓

Denoted by adj A.

$$A\,(\text{adj}\,A) = (\text{adj}\,A)\,A = |A|\,I$$

$|A| = 0 \Rightarrow A$ is singular

$|A| \neq 0 \Rightarrow A$ is non-singular

A and B nonsingular matrices of the same order $\Rightarrow$ AB and BA also are nonsingular matrices of the same order.

$$|AB| = |A|\,|B|.$$

$$(\text{adj}\,A)\,A = |A|\,I \Rightarrow |(\text{adj}\,A)|\,|A| = \begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix} = |A|^3 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow |(\text{adj}\,A)|\,|A| = |A|^3\,(1)$$

$$\Rightarrow |(\text{adj}\,A)| = |A|^2$$

In general, $|\text{adj}\,(A)| = |A|^{n-1}$, n is the order of A

A square matrix is invertible if and only if it is non singular matrix.