

Question 13: Which of the following functions have a finite number of points of discontinuity in \mathbb{R} ([.] represents the greatest integer function)?

- A) $\tan x$
- B) $x[x]$
- C) $|x| / x$
- D) $\sin[\pi x]$

Solution:

$f(x) = \tan x$ is discontinuous when $x = (2n + 1) \pi / 2, n \in \mathbb{Z}$

$f(x) = x[x]$ is discontinuous when $x = k, k \in \mathbb{Z}$

$f(x) = \sin [n\pi x]$ is discontinuous when $n\pi x = k, k \in \mathbb{Z}$

Thus, all the above functions have an infinite number of points of discontinuity. But, if $(x) = |x| / x$ is discontinuous when $x = 0$ only.

Question 14: The number of values of $x \in [0, 2]$ at which $f(x) = |x - [1/2]| + |x - 1| + \tan x$ is not differentiable is

- A) 0
- B) 1
- C) 3
- D) None of these

Solution:

$|x - [1/2]|$ is continuous everywhere but not differentiable at $x = 1/2$, $|x - 1|$ is continuous everywhere, but not differentiable at $x = 1$ and $\tan x$ is continuous in $[0, 2]$ except at $x = \pi/2$. Hence, $f(x)$ is not differentiable at $x = 1/2, 1, \pi/2$.

Question 15:

$$\lim_{x \rightarrow \pi/2} (\sec \theta - \tan \theta) =$$

Solution:

$$\lim_{\theta \rightarrow \pi/2} \frac{1 - \sin \theta}{\cos \theta} = \lim_{\theta \rightarrow \pi/2} \frac{(\cos \frac{\theta}{2} - \sin \frac{\theta}{2})^2}{(\cos \frac{\theta}{2} - \sin \frac{\theta}{2})(\cos \frac{\theta}{2} + \sin \frac{\theta}{2})} = 0$$