

Solution:

For continuous

$$\begin{aligned}\lim_{x \rightarrow 2} f(x) &= f(2) = k \\ \Rightarrow k &= \lim_{x \rightarrow 2} \frac{x^3 + x^2 - 16x + 20}{(x-2)^2} \\ &= \lim_{x \rightarrow 2} \frac{(x^2 - 4x + 4)(x+5)}{(x-2)^2} \\ &= 7.\end{aligned}$$

Question 12:

$$f(x) = \begin{cases} \frac{x^2 - 4x + 3}{x^2 - 1} & \text{for } x \neq 1 \\ 2 & \text{for } x = 1 \end{cases}$$

, then find the condition for the function to be continuous or discontinuous.

Solution:

$$f(x) = \left\{ \frac{x^2 - 4x + 3}{x^2 - 1} \right\}$$

for $x = 1$

$$f(1) = 2,$$

$$\begin{aligned}f(1+) &= \lim_{x \rightarrow 1+} \frac{x^2 - 4x + 3}{x^2 - 1} \\ &= \lim_{x \rightarrow 1+} \frac{(x-3)}{(x+1)} \\ &= -1\end{aligned}$$

$$\begin{aligned}f(1-) &= \lim_{x \rightarrow 1-} \frac{x^2 - 4x + 3}{x^2 - 1} \\ &= -1 \\ \Rightarrow f(1) &\neq f(1-)\end{aligned}$$

Hence, the function is discontinuous at $x = 1$.

Question 13: Which of the following functions have a finite number of points of discontinuity in \mathbb{R} ([.] represents the greatest integer function)?

A) $\tan x$