$= e^2$

$[\because \ \lim_{x\to 0} \ (1+x)^{1/x} = e]$

Question 9: Solve

$\lim_{x\to 0} \ \frac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$

Solution:

$\lim_{x\to 0} \ \frac{x\tan 2x - 2x\tan x}{(1-\cos \ 2x)^2}$
$= \lim_{x\to 0} \ \frac{x(\tan \ 2x - 2\tan x)}{(2\sin^2 x)^2}$
$= \lim_{x\to 0} \ \frac{1}{4} \ \frac{x\,(\tan 2x - 2\tan x)}{\sin^4 x}$
$= \lim_{x\to 0} \ \frac{1}{4} \frac{x\left\{\left(2x + \frac{1}{3}(2x)^3 + \frac{2}{15}\ (2x)^5 + \ldots\right) - 2\left(x + \frac{x^3}{3} + \frac{2}{15}x^5 + \ldots\right)\right\}}{x^4\left(1 - \frac{x^2}{3\,!} + \frac{x^4}{5\,!} + \ldots.\right)^4}$
$= \frac{1}{4} \cdot \left(\frac{8}{3} - \frac{2}{3}\right)$
$= \frac{2}{4}$
$= \frac{1}{2}.$

Question 10: The function

$f(x) = \frac{\log(1+ax) - \log(1-bx)}{x}$
is not defined at x = 0. The value which should be assigned to f at x = 0 so that it is continuous at x = 0, is

Solution:

Since limit of a function is a + b as x → 0, therefore to be continuous at a function, its value must be a + b at x = 0

⇒ f (0) = a + b

Question 11: Evaluate

$$f(x) = \{ \begin{matrix} \frac{x^3 + x^2 - 16x + 20}{(x-2)^2} & \text{if } x \neq 2 \\ k & \text{if } x = 2 \end{matrix}$$

Solution: