$$\lim_{x\to 0} \frac{\sqrt{\frac{1}{2}(1-\cos 2x)}}{x} = \lim_{x\to 0} \frac{|\sin x|}{x}$$

So,

$$\lim_{x\to 0+} \frac{|\sin x|}{x} = 1$$

and

$$\lim_{x\to 0-} \frac{|\sin x|}{x} = -1$$

Hence, the limit doesn't exist.

Question 7: Solve

$$\lim_{x\to 0} \left\{\tan\left(\frac{\pi}{4}+x\right)\right\}^{1/x}$$

Solution:

Given limit =

$$= \lim_{x\to 0} \left(\frac{1+\tan x}{1-\tan x}\right)^{1/x}$$

$$= \lim_{x\to 0} \frac{\{(1+\tan x)^{1/\tan x}\}^{(\tan x)/x}}{\{(1-\tan x)^{1/\tan x}\}^{(\tan x)/x}}$$

$$= \frac{e}{e^{-1}}$$

$$= e^2.$$

Question 8: Solve

$$\lim_{x\to 0} \left(\frac{1+5x^2}{1+3x^2}\right)^{1/x^2}$$

Solution:

$$\lim_{x\to 0} \left(\frac{1+5x^2}{1+3x^2}\right)^{1/x^2}$$

$$= \frac{\lim_{x\to 0}\left[(1+5x^2)^{1/5x^2}\right]^5}{\lim_{x\to 0}\left[(1+3x^2)^{1/3x^2}\right]^3}$$

$$= \frac{e^5}{e^3}$$

$$= e^2$$