Example 23 Find the approximate value of $f(3.02)$, where $f(x) = 3x^2 + 5x + 3$.

Solution Let $x = 3$ and $\Delta x = 0.02$. Then

$$f(3.\ 02) = f(x + \Delta x) = 3(x + \Delta x)^2 + 5(x + \Delta x) + 3$$

Note that $\Delta y = f(x + \Delta x) - f(x)$. Therefore

$$f(x + \Delta x) = f(x) + \Delta y$$

$$\approx f(x) + f'(x)\ \Delta x \qquad (\text{as } dx = \Delta x)$$

or $$f(3.02) \approx (3x^2 + 5x + 3) + (6x + 5)\ \Delta x$$

$$= (3(3)^2 + 5(3) + 3) + (6(3) + 5)\ (0.02) \quad (\text{as } x = 3, \Delta x = 0.02)$$

$$= (27 + 15 + 3) + (18 + 5)\ (0.02)$$

$$= 45 + 0.46 = 45.46$$

Hence, approximate value of $f(3.02)$ is 45.46.

Example 24 Find the approximate change in the volume V of a cube of side x meters caused by increasing the side by 2%.

Solution Note that

$$\mathrm{V} = x^3$$

or $$d\mathrm{V} = \left(\frac{d\mathrm{V}}{dx}\right)\Delta x = (3x^2)\ \Delta x$$

$$= (3x^2)\ (0.02x) = 0.06x^3\ \mathrm{m}^3 \qquad (\text{as } 2\% \text{ of } x \text{ is } 0.02x)$$

Thus, the approximate change in volume is $0.06\ x^3\ \mathrm{m}^3$.

Example 25 If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its volume.

Solution Let r be the radius of the sphere and Δr be the error in measuring the radius. Then $r = 9$ cm and $\Delta r = 0.03$ cm. Now, the volume V of the sphere is given by

$$\mathrm{V} = \frac{4}{3}\pi r^3$$

or $$\frac{d\mathrm{V}}{dr} = 4\pi r^2$$

Therefore $$d\mathrm{V} = \left(\frac{d\mathrm{V}}{dr}\right)\Delta r = (4\pi r^2)\Delta r$$

$$= 4\pi(9)^2\ (0.03) = 9.72\pi\ \mathrm{cm}^3$$

Thus, the approximate error in calculating the volume is $9.72\pi\ \mathrm{cm}^3$.