or
$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{-3b\cos^2 t\sin t}{3a\sin^2 t\cos t}=\frac{-b}{a}\frac{\cos t}{\sin t}$$

Therefore, slope of the tangent at $t=\frac{\pi}{2}$ is

$$\left.\frac{dy}{dx}\right]_{t=\frac{\pi}{2}}=\frac{-b\cos\frac{\pi}{2}}{a\sin\frac{\pi}{2}}=0$$

Also, when $t=\frac{\pi}{2}$, $x = a$ and $y = 0$. Hence, the equation of tangent to the given curve at $t=\frac{\pi}{2}$, i.e., at $(a, 0)$ is

$$y - 0 = 0\,(x - a),\text{ i.e., } y = 0.$$

EXERCISE 6.3

1. Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at $x = 4$.
2. Find the slope of the tangent to the curve $y=\frac{x-1}{x-2},\ x\neq 2$ at $x = 10$.
3. Find the slope of the tangent to curve $y = x^3 - x + 1$ at the point whose x-coordinate is 2.
4. Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose x-coordinate is 3.
5. Find the slope of the normal to the curve $x=a\cos^3\theta,\ y=a\sin^3\theta$ at $\theta=\frac{\pi}{4}$.
6. Find the slope of the normal to the curve $x=1-a\sin\theta, y=b\cos^2\theta$ at $\theta=\frac{\pi}{2}$.
7. Find points at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$ is parallel to the x-axis.
8. Find a point on the curve $y = (x - 2)^2$ at which the tangent is parallel to the chord joining the points (2, 0) and (4, 4).

9. Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$.

10. Find the equation of all lines having slope -1 that are tangents to the curve $y = \frac{1}{x-1}$, $x \neq 1$.

11. Find the equation of all lines having slope 2 which are tangents to the curve $y = \frac{1}{x-3}$, $x \neq 3$.

12. Find the equations of all lines having slope 0 which are tangent to the curve $y = \frac{1}{x^2 - 2x + 3}$.

13. Find points on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ at which the tangents are

(i) parallel to x-axis (ii) parallel to y-axis.

14. Find the equations of the tangent and normal to the given curves at the indicated points:

(i) $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at (0, 5)

(ii) $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at (1, 3)

(iii) $y = x^3$ at (1, 1)

(iv) $y = x^2$ at (0, 0)

(v) $x = \cos t$, $y = \sin t$ at $t = \frac{\pi}{4}$

15. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is

(a) parallel to the line $2x - y + 9 = 0$

(b) perpendicular to the line $5y - 15x = 13$.

16. Show that the tangents to the curve $y = 7x^3 + 11$ at the points where $x = 2$ and $x = -2$ are parallel.

17. Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to the y-coordinate of the point.

18. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

19. Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which the tangents are parallel to the x-axis.

20. Find the equation of the normal at the point (am^2, am^3) for the curve $ay^2 = x^3$.

21. Find the equation of the normals to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$.

22. Find the equations of the tangent and normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$.

23. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles* if $8k^2 = 1$.

24. Find the equations of the tangent and normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x_0, y_0).

25. Find the equation of the tangent to the curve $y = \sqrt{3x - 2}$ which is parallel to the line $4x - 2y + 5 = 0$.

Choose the correct answer in Exercises 26 and 27.

26. The slope of the normal to the curve $y = 2x^2 + 3 \sin x$ at $x = 0$ is

(A) 3 (B) $\frac{1}{3}$ (C) –3 (D) $-\frac{1}{3}$

27. The line $y = x + 1$ is a tangent to the curve $y^2 = 4x$ at the point

(A) (1, 2) (B) (2, 1) (C) (1, – 2) (D) (– 1, 2)

6.5 Approximations

In this section, we will use differentials to approximate values of certain quantities.

Let $f : D \rightarrow \mathbf{R}$, $D \subset \mathbf{R}$, be a given function and let $y = f(x)$. Let Δx denote a small increment in x. Recall that the increment in y corresponding to the increment in x, denoted by Δy, is given by $\Delta y = f(x + \Delta x) - f(x)$. We define the following

(i) The differential of x, denoted by dx, is defined by $dx = \Delta x$.

(ii) The differential of y, denoted by dy, is defined by $dy = f'(x)\, dx$ or $dy = \left(\frac{dy}{dx}\right)\Delta x$.

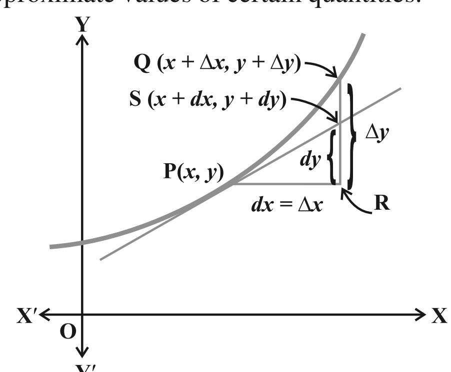

Fig 6.8

* Two curves intersect at right angle if the tangents to the curves at the point of intersection are perpendicular to each other.