Q42. The points at which the tangents to the curve $y = x^3 - 12x + 18$ are parallel to x-axis are:

(*a*) (2, – 2), (– 2, – 34) (*b*) (2, 34), (– 2, 0)

(*c*) (0, 34), (– 2, 0) (*d*) (2, 2), (– 2, 34)

Sol. Given that y = $x^3 - 12x + 18$

Differentiating both sides w.r.t. x, we have

$$\Rightarrow \quad \frac{dy}{dx} = 3x^2 - 12$$

Since the tangents are parallel to x-axis, then $\frac{dy}{dx} = 0$

$$\therefore \quad 3x^2 - 12 = 0 \quad \Rightarrow \quad x = \pm 2$$

$$\therefore \quad y_{x=2} = (2)^3 - 12(2) + 18 = 8 - 24 + 18 = 2$$

$$y_{x=-2} = (-2)^3 - 12(-2) + 18 = -8 + 24 + 18 = 34$$

$\therefore$ Points are (2, 2) and (– 2, 34)

Hence, the correct option is (d).

Q43. The tangent to the curve $y = e^{2x}$ at the point (0, 1) meets x-axis at:

(*a*) (0, 1) (*b*) $\left(-\frac{1}{2}, 0\right)$ (*c*) (2, 0) (*d*) (0, 2)

Sol. Equation of the curve is $y = e^{2x}$

Slope of the tangent $\frac{dy}{dx} = 2e^{2x} \Rightarrow \left.\frac{dy}{dx}\right|_{(0,1)} = 2 \cdot e^0 = 2$

$\therefore$ Equation of tangent to the curve at (0, 1) is

$$y - 1 = 2(x - 0)$$

$$\Rightarrow \quad y - 1 = 2x \quad \Rightarrow y - 2x = 1$$

Since the tangent meets x-axis where $y = 0$

$$\therefore \quad 0 - 2x = 1 \quad \Rightarrow \quad x = \frac{-1}{2}$$

So the point is $\left(-\frac{1}{2}, 0\right)$

Hence, the correct option is (b).

Q44. The slope of tangent to the curve $x = t^2 + 3t - 8$ and $y = 2t^2 - 2t - 5$ at the point (2, – 1) is:

(*a*) $\frac{22}{7}$ (*b*) $\frac{6}{7}$ (*c*) $-\frac{6}{7}$ (*d*) – 6

Sol. The given curve is $x = t^2 + 3t - 8$ and $y = 2t^2 - 2t - 5$

$$\frac{dx}{dt} = 2t + 3 \quad \text{and} \quad \frac{dy}{dt} = 4t - 2$$

$$\therefore \quad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4t - 2}{2t + 3}$$

Now (2, – 1) lies on the curve

$\therefore \quad 2 = t^2 + 3t - 8 \Rightarrow t^2 + 3t - 10 = 0$

$\Rightarrow t^2 + 5t - 2t - 10 = 0$

$\Rightarrow t(t + 5) - 2(t + 5) = 0$

$\Rightarrow (t + 5)(t - 2) = 0$

$\therefore t = 2, t = -5$ and $-1 = 2t^2 - 2t - 5$

$\Rightarrow \quad 2t^2 - 2t - 4 = 0$

$\Rightarrow \quad t^2 - t - 2 = 0 \Rightarrow t^2 - 2t + t - 2 = 0$

$\Rightarrow \quad t(t - 2) + 1(t - 2) = 0 \Rightarrow (t + 1)(t - 2) = 0$

$\Rightarrow \quad t = -1$ and $t = 2$

So t = 2 is common value

$$\therefore \quad \text{Slope } \frac{dy}{dx}_{x=2} = \frac{4 \times 2 - 2}{2 \times 2 + 3} = \frac{6}{7}$$

Hence, the correct option is (b).

Q45. The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$ intersect at an angle of:

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$

Sol. The given curves are $x^3 - 3xy^2 + 2 = 0$ ...(i)

and $3x^2y - y^3 - 2 = 0$ ...(ii)

Differentiating eq. (i) w.r.t. x, we get

$$3x^2 - 3\left(x \cdot 2y\frac{dy}{dx} + y^2 \cdot 1\right) = 0$$

$$\Rightarrow \quad x^2 - 2xy\frac{dy}{dx} - y^2 = 0 \Rightarrow 2xy\frac{dy}{dx} = x^2 - y^2$$

$$\therefore \quad \frac{dy}{dx} = \frac{x^2 - y^2}{2xy}$$

So slope of the curve $m_1 = \frac{x^2 - y^2}{2xy}$

Differentiating eq. (ii) w.r.t. x, we get

$$3\left[x^2\frac{dy}{dx} + y \cdot 2x\right] - 3y^2 \cdot \frac{dy}{dx} = 0$$

$$x^2\frac{dy}{dx} + 2xy - y^2\frac{dy}{dx} = 0 \Rightarrow (x^2 - y^2)\frac{dy}{dx} = -2xy$$

$$\therefore \quad \frac{dy}{dx} = \frac{-2xy}{x^2 - y^2}$$

So the slope of the curve $m_2 = \dfrac{-2xy}{x^2 - y^2}$

Now $m_1 \times m_2 = \dfrac{x^2 - y^2}{2xy} \times \dfrac{-2xy}{x^2 - y^2} = -1$

So the angle between the curves is $\dfrac{\pi}{2}$.

Hence, the correct option is (*c*).

Q46. The interval on which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing is:

(*a*) $[-1, \infty)$ (*b*) $[-2, -1]$

(*c*) $(-\infty, -2]$ (*d*) $[-1, 1]$

Sol. The given function is $f(x) = 2x^3 + 9x^2 + 12x - 1$

$$f'(x) = 6x^2 + 18x + 12$$

For increasing and decreasing $f'(x) = 0$

$\therefore \quad 6x^2 + 18x + 12 = 0$

$\Rightarrow \quad x^2 + 3x + 2 = 0 \quad \Rightarrow x^2 + 2x + x + 2 = 0$

$\Rightarrow \quad x(x + 2) + 1(x + 2) = 0 \quad \Rightarrow (x + 2)(x + 1) = 0$

$\Rightarrow \quad x = -2, x = -1$

The possible intervals are $(-\infty, -2), (-2, -1), (-1, \infty)$

Now $\quad f'(x) = (x + 2)(x + 1)$

$\Rightarrow \quad f'(x)_{(-\infty, -2)} = (-)(-) = (+)$ increasing

$\Rightarrow \quad f'(x)_{(-2, -1)} = (+)(-) = (-)$ decreasing

$\Rightarrow \quad f'(x)_{(-1, \infty)} = (+)(+) = (+)$ increasing

Hence, the correct option is (*b*).

Q47. Let the $f : \mathbf{R} \to \mathbf{R}$ be defined by $f(x) = 2x + \cos x$, then f:

(*a*) has a minimum at $x = \pi$ (*b*) has a maximum at $x = 0$

(*c*) is a decreasing function (*d*) is an increasing function

Sol. Given that $\quad f(x) = 2x + \cos x$

$\quad f'(x) = 2 - \sin x$

Since $\quad f'(x) > 0 \; \forall \; x$

So $f(x)$ is an increasing function.

Hence, the correct option is (*d*).

Q48. $y = x(x - 3)^2$ decreases for the values of x given by:

(*a*) $1 < x < 3$ (*b*) $x < 0$ (*c*) $x > 0$ (*d*) $0 < x < \dfrac{3}{2}$

Sol. Here $y = x(x - 3)^2$

$$\frac{dy}{dx} = x \cdot 2(x - 3) + (x - 3)^2 \cdot 1 \quad \Rightarrow \frac{dy}{dx} = 2x(x - 3) + (x - 3)^2$$

For increasing and decreasing $\frac{dy}{dx} = 0$

$\therefore \quad 2x(x-3) + (x-3)^2 = 0 \quad \Rightarrow (x-3)(2x+x-3) = 0$

$\Rightarrow \quad (x-3)(3x-3) = 0 \quad \Rightarrow 3(x-3)(x-1) = 0$

$\therefore \quad x = 1, 3$

$\therefore$ Possible intervals are $(-\infty, 1), (1, 3), (3, \infty)$

$$\frac{dy}{dx} = (x-3)(x-1)$$

For $(-\infty, 1) = (-)(-) = (+)$ increasing

For $(1, 3) = (-)(+) = (-)$ decreasing

For $(3, \infty) = (+)(+) = (+)$ increasing

So the function decreases in $(1, 3)$ or $1 < x < 3$

Hence, the correct option is (*a*).

Q49. The function $f(x) = 4\sin^3 x - 6\sin^2 x + 12\sin x + 100$ is strictly

(*a*) increasing in $\left(\pi, \frac{3\pi}{2}\right)$ (*b*) decreasing in $\left(\frac{\pi}{2}, \pi\right)$

(*c*) decreasing in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (*d*) decreasing in $\left[0, \frac{\pi}{2}\right]$

Sol. Here,

$$f(x) = 4\sin^3 x - 6\sin^2 x + 12\sin x + 100$$

$$f'(x) = 12\sin^2 x \cdot \cos x - 12\sin x\cos x + 12\cos x$$

$$= 12\cos x\,[\sin^2 x - \sin x + 1]$$

$$= 12\cos x\,[\sin^2 x + (1 - \sin x)]$$

$\because \quad 1 - \sin x \geq 0$ and $\sin^2 x \geq 0$

$\therefore \quad \sin^2 x + 1 - \sin x \geq 0$ (when $\cos x > 0$)

Hence, $f'(x) > 0$, when $\cos x > 0$ i.e., $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

So, $f(x)$ is increasing where $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ and $f'(x) < 0$

when $\cos x < 0$ i.e. $x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

Hence, $f(x)$ is decreasing when $x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

As $\left(\frac{\pi}{2}, \pi\right) \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

So $f(x)$ is decreasing in $\left(\frac{\pi}{2}, \pi\right)$

Hence, the correct option is (*b*).