Q9. A swimming pool is to be drained for cleaning. If L represents the number of litres of water in the pool t seconds after the pool has been plugged off to drain and $L = 200(10 - t)^2$. How fast is the water running out at the end of 5 seconds? What is the average rate at which the water flows out during the first 5 seconds?

Sol. Given that $L = 200(10 - t)^2$

where L represents the number of litres of water in the pool.

Differentiating both sides w.r.t, t, we get

$$\frac{dL}{dt} = 200 \times 2(10 - t)(-1) = -400(10 - t)$$

But the rate at which the water is running out

$$= -\frac{dL}{dt} = 400(10 - t) \quad ...(1)$$

Rate at which the water is running after 5 seconds

$$= 400 \times (10 - 5) = 2000 \text{ L/s (final rate)}$$

For initial rate put $t = 0$

$$= 400(10 - 0) = 4000 \text{ L/s}$$

The average rate at which the water is running out

$$= \frac{\text{Initial rate} + \text{Final rate}}{2} = \frac{4000 + 2000}{2} = \frac{6000}{2} = 3000 \text{ L/s}$$

Hence, the required rate = 3000 L/s.

Q10. The volume of a cube increases at a constant rate. Prove that the increase in its surface area varies inversely as the length of the side.

Sol. Let x be the length of the cube

$\therefore$ Volume of the cube $V = x^3$ ...(1)

Given that $\frac{dV}{dt} = K$

Differentiating Eq. (1) w.r.t. t, we get

$$\frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt} = K \text{ (constant)}$$

$$\therefore \quad \frac{dx}{dt} = \frac{K}{3x^2}$$

Now surface area of the cube, $S = 6x^2$

Differentiating both sides w.r.t. t, we get

$$\frac{ds}{dt} = 6 \cdot 2 \cdot x \cdot \frac{dx}{dt} = 12x \cdot \frac{K}{3x^2}$$

$$\Rightarrow \quad \frac{ds}{dt} = \frac{4K}{x} \Rightarrow \frac{ds}{dt} \propto \frac{1}{x} \quad (4K = \text{constant})$$

Hence, the surface area of the cube varies inversely as the length of the side.