Q6. Find the approximate value of $(1.999)^5$.

Sol. $(1.999)^5 = (2 - 0.001)^5$

Let $x = 2$ and $\Delta x = -0.001$

Let $y = x^5$

Differentiating both sides w.r.t, x, we get

$$\frac{dy}{dx} = 5x^4 = 5(2)^4 = 80$$

Now $\Delta y = \left(\frac{dy}{dx}\right) \cdot \Delta x = 80 \cdot (-0.001) = -0.080$

$\therefore$ $(1.999)^5 = y + \Delta y$

$= x^5 - 0.080 = (2)^5 - 0.080 = 32 - 0.080 = 31.92$

Hence, approximate value of $(1.999)^5$ is 31.92.

Q7. Find the approximate volume of metal in a hollow spherical shell whose internal and external radii are 3 cm and 3.0005 cm respectively.

Sol. Internal radius $r = 3$ cm

and external radius $R = r + \Delta r = 3.0005$ cm

$\therefore$ $\Delta r = 3.0005 - 3 = 0.0005$ cm

Let $y = r^3 \Rightarrow y + \Delta y = (r + \Delta r)^3 = R^3 = (3.0005)^3$ ...(*i*)

Differentiating both sides w.r.t., r, we get

$$\frac{dy}{dr} = 3r^2$$

$\therefore$ $\Delta y = \frac{dy}{dr} \times \Delta r = 3r^2 \times 0.0005$

$= 3 \times (3)^2 \times 0.0005 = 27 \times 0.0005 = 0.0135$

$\therefore$ $(3.0005)^3 = y + \Delta y$ [From eq. (*i*)]

$= (3)^3 + 0.0135 = 27 + 0.0135 = 27.0135$

Volume of the shell $= \frac{4}{3}\pi[R^3 - r^3]$

$= \frac{4}{3}\pi\,[27.0135 - 27] = \frac{4}{3}\pi \times 0.0135$

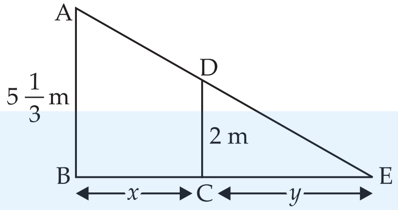
$= 4\pi \times 0.005 = 4 \times 3.14 \times 0.0045 = 0.018\,\pi\,\text{cm}^3$

Hence, the approximate volume of the metal in the shell is 0.018π cm^3.

Q8. A man, 2m tall, walks at the rate of $1\frac{2}{3}$ m/s towards a street light which is $5\frac{1}{3}$ m above the ground. At what rate is the tip of his shadow moving? At what rate is the length of the shadow changing when he is $3\frac{1}{3}$ m from the base of the light?

Sol. Let AB is the height of street light post and CD is the height of the man such that

$AB = 5\frac{1}{3} = \frac{16}{3}$ m and CD = 2 m



Let BC = x length (the distance of the man from the lamp post) and CE = y is the length of the shadow of the man at any instant.

From the figure, we see that

$\Delta ABE \sim \Delta DCE$ [by AAA Similarity]

$\therefore$ Taking ratio of their corresponding sides, we get

$$\frac{AB}{CD} = \frac{BE}{CE} \Rightarrow \frac{AB}{CD} = \frac{BC + CE}{CE}$$

$$\Rightarrow \frac{16/3}{2} = \frac{x+y}{y} \Rightarrow \frac{8}{3} = \frac{x+y}{y}$$

$$\Rightarrow 8y = 3x + 3y \Rightarrow 8y - 3y = 3x \Rightarrow 5y = 3x$$

Differentiating both sides w.r.t, t, we get

$$\cdot\frac{dy}{dt} = 3 \cdot \frac{dx}{dt}$$

$$\Rightarrow \frac{dy}{dt} = \frac{3}{5} \cdot \frac{dx}{dt} \Rightarrow \frac{dy}{dt} = \frac{3}{5} \cdot \left(-1\frac{2}{3}\right) = \frac{3}{5} \cdot \left(\frac{-5}{3}\right)$$

[$\because$ man is moving in opposite direction]

$= -1$ m/s

Hence, the length of shadow is decreasing at the rate of 1 m/s.

Now let $u = x + y$

(u = distance of the tip of shadow from the light post)

Differentiating both sides w.r.t. t, we get

$$\frac{du}{dt} = \frac{dx}{dt} + \frac{dy}{dt}$$

$$= \left(-1\frac{2}{3} - 1\right) = -\left(\frac{5}{3} + 1\right) = -\frac{8}{3} = -2\frac{2}{3} \text{ m/s}$$

Hence, the tip of the shadow is moving at the rate of $2\frac{2}{3}$ m/s towards the light post and the length of shadow decreasing at the rate of 1 m/s.