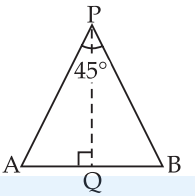
Q4. Two men A and B start with velocities V at the same time from the junction of two roads inclined at 45° to each other. If they travel by different roads, find the rate at which they are being separated.



Sol. Let P be any point at which the two roads are inclined at an angle of 45°.
Two men A and B are moving along the roads PA and PB respectively with the same speed 'V'.
Let A and B be their final positions such that
AB = y
$\angle APB = 45°$ and they move with the same speed.
$\therefore$ ΔAPB is an isosceles triangle. Draw $PQ \perp AB$

$AB = y \quad \therefore \quad AQ = \frac{y}{2}$ and $PA = PB = x$ (let)

$$\angle APQ = \angle BPQ = \frac{45}{2} = 22\frac{1}{2}°$$

[$\because$ In an isosceles Δ, the altitude drawn from the vertex, bisects the base]
Now in right ΔAPQ,

$$\sin 22\frac{1}{2}° = \frac{AQ}{AP}$$

$$\Rightarrow \quad \sin 22\frac{1}{2}° = \frac{\frac{y}{2}}{x} = \frac{y}{2x} \quad \Rightarrow y = 2x \cdot \sin 22\frac{1}{2}°$$

Differentiating both sides w.r.t, t, we get

$$\frac{dy}{dt} = 2 \cdot \frac{dx}{dt} \cdot \sin 22\frac{1}{2}°$$

$$= 2 \cdot V \cdot \frac{\sqrt{2-\sqrt{2}}}{2} \quad \left[\because \sin 22\frac{1}{2}° = \frac{\sqrt{2-\sqrt{2}}}{2}\right]$$

$$= \sqrt{2-\sqrt{2}}\ V \text{ m/s}$$

Hence, the rate of their separation is $\sqrt{2-\sqrt{2}}$ V unit/s.

Q5. Find an angle θ, $0 < \theta < \frac{\pi}{2}$, which increases twice as fast as its sine.

Sol. As per the given condition,

$$\frac{d\theta}{dt} = 2\frac{d}{dt}(\sin\theta)$$

$$\Rightarrow \quad \frac{d\theta}{dt} = 2\cos\theta \cdot \frac{d\theta}{dt} \quad \Rightarrow 1 = 2\cos\theta$$

$$\therefore \quad \cos\theta = \frac{1}{2} \quad \Rightarrow \quad \cos\theta = \cos\frac{\pi}{3} \quad \Rightarrow \theta = \frac{\pi}{3}$$

Hence, the required angle is $\frac{\pi}{3}$.