6.3 EXERCISE

SHORT ANSWER TYPE QUESTIONS

Q1. A spherical ball of salt is dissolving in water in such a manner that the rate of decrease of the volume at any instant is proportional to the surface. Prove that the radius is decreasing at a constant rate.

Sol. Ball of salt is spherical

$\therefore$ Volume of ball, $V = \frac{4}{3}\pi r^3$, where r = radius of the ball

As per the question, $\frac{dV}{dt} \propto S$, where S = surface area of the ball

$$\Rightarrow \quad \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) \propto 4\pi r^2 \qquad [\because \quad S = 4\pi r^2]$$

$$\Rightarrow \quad \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt} \propto 4\pi r^2$$

$$\Rightarrow \quad 4\pi r^2 \cdot \frac{dr}{dt} = K \cdot 4\pi r^2 \qquad \text{(K = Constant of proportionality)}$$

$$\Rightarrow \quad \frac{dr}{dt} = K \cdot \frac{4\pi r^2}{4\pi r^2}$$

$$\therefore \quad \frac{dr}{dt} = K \cdot 1 = K$$

Hence, the radius of the ball is decreasing at constant rate.

Q2. If the area of a circle increases at a uniform rate, then prove that perimeter varies inversely as the radius.

Sol. We know that:

Area of circle, $A = \pi r^2$, where r = radius of the circle.

and perimeter = $2\pi r$

As per the question,

$$\frac{dA}{dt} = K, \text{ where K = constant}$$

$$\Rightarrow \quad \frac{d}{dt}(\pi r^2) = K \quad \Rightarrow \quad \pi \cdot 2r \cdot \frac{dr}{dt} = K$$

$$\therefore \quad \frac{dr}{dt} = \frac{K}{2\pi r} \qquad ...(1)$$

Now Perimeter $\quad c = 2\pi r$

Differentiating both sides w.r.t., t, we get

$$\Rightarrow \quad \frac{dc}{dt} = \frac{d}{dt}(2\pi r) \quad \Rightarrow \quad \frac{dc}{dt} = 2\pi \cdot \frac{dr}{dt}$$

$$\Rightarrow \quad \frac{dc}{dt} = 2\pi \cdot \frac{K}{2\pi r} = \frac{K}{r} \qquad \text{[From (1)]}$$

$$\Rightarrow \quad \frac{dc}{dt} \propto \frac{1}{r}$$

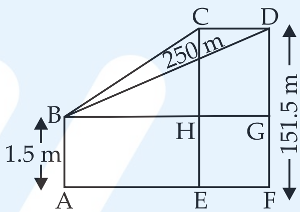
Hence, the perimeter of the circle varies inversely as the radius of the circle.

Q3. A kite is moving horizontally at a height of 151.5 metres. If the speed of the kite is 10 m/s, how fast is the string being let out; when the kite is 250 m away from the boy who is flying the kite? The height of the boy is 1.5 m.

Sol. Given that height of the kite (h) = 151.5 m

Speed of the kite(V) = 10 m/s

Let FD be the height of the kite and AB be the height of the boy.



Let AF = x m

$\therefore$ BG = AF = x m

and $\frac{dx}{dt} = 10$ m/s

From the figure, we get that

$$GD = DF - GF \Rightarrow DF - AB$$

$$= (151.5 - 1.5) \text{ m} = 150 \text{ m} \qquad [\because \; AB = GF]$$

Now in ΔBGD,

$$BG^2 + GD^2 = BD^2 \qquad \text{(By Pythagoras Theorem)}$$

$$\Rightarrow \quad x^2 + (150)^2 = (250)^2$$

$$\Rightarrow \quad x^2 + 22500 = 62500 \quad \Rightarrow x^2 = 62500 - 22500$$

$$\Rightarrow \quad x^2 = 40000 \quad \Rightarrow x = 200 \text{ m}$$

Let initially the length of the string be y m

$\therefore$ In ΔBGD

$$BG^2 + GD^2 = BD^2 \quad \Rightarrow x^2 + (150)^2 = y^2$$

Differentiating both sides w.r.t., t, we get

$$\Rightarrow \quad 2x \cdot \frac{dx}{dt} + 0 = 2y \cdot \frac{dy}{dt} \qquad \left[\because \frac{dx}{dt} = 10 \text{ m/s}\right]$$

$$\Rightarrow \quad 2 \times 200 \times 10 = 2 \times 250 \times \frac{dy}{dt}$$

$$\therefore \quad \frac{dy}{dt} = \frac{2 \times 200 \times 10}{2 \times 250} = 8 \text{ m/s}$$

Hence, the rate of change of the length of the string is 8 m/s.