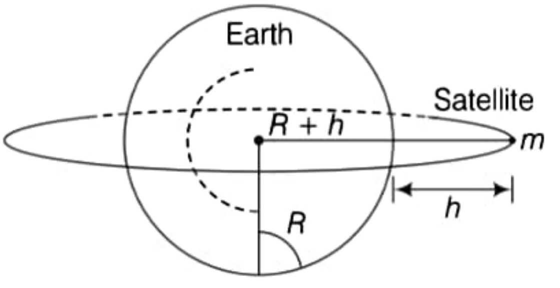
Q. 36 A satellite is to be placed in equatorial geostationary orbit around the earth for communication.

(a) Calculate height of such a satellite.

(b) Find out the minimum number of satellites that are needed to cover entire earth, so that atleast one satellite is visible from any point on the equator.



$[M = 6 \times 10^{24}$ kg, $R = 6400$ km, $T = 24$ h, $G = 6.67 \times 10^{-11}$ SI unit]

Ans. Consider the adjacent diagram

Given, mass of the earth $M = 6 \times 10^{24}$ kg

Radius of the earth, $R = 6400 \text{ km} = 6.4 \times 10^6$ m

Time period $T = 24$ h

$= 24 \times 60 \times 60 = 86400$ s

$G = 6.67 \times 10^{-11} \text{ N-m}^2/\text{kg}^2$

(a) Time period
$$T = 2\pi\sqrt{\frac{(R+h)^3}{GM}} \qquad \left[\because v_o = \sqrt{\frac{GM}{R+h}} \text{ and } T = \frac{2\pi(R+h)}{v_o}\right]$$

$$\Rightarrow \quad T^2 = 4\pi^2 \frac{(R+h)^3}{GM} \Rightarrow (R+h)^3 = \frac{T^2GM}{4\pi^2}$$

$$\Rightarrow \quad R + h = \left(\frac{T^2GM}{4\pi^2}\right)^{1/3} \Rightarrow h = \left(\frac{T^2GM}{4\pi^2}\right)^{1/3} - R$$

$$\Rightarrow \quad h = \left[\frac{(24 \times 60 \times 60)^2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{4 \times (3.14)^2}\right]^{1/3} - 6.4 \times 10^6$$

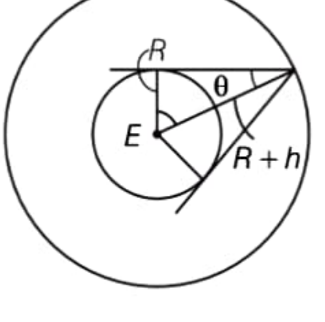
$= 4.23 \times 10^7 - 6.4 \times 10^6$

$= (42.3 - 6.4) \times 10^6$

$= 35.9 \times 10^6$ m

$= 3.59 \times 10^7$ m

(b) If satellite is at height h from the earth's surface, then according to the diagram.



$$\cos\theta = \frac{R}{R+h} = \frac{1}{\left(1+\frac{h}{R}\right)} = \frac{1}{\left(1+\frac{3.59\times10^7}{6.4\times10^6}\right)}$$

$$= \frac{1}{1+5.61} = \frac{1}{6.61} = 0.1513 = \cos 81°18'$$

$$\theta = 81°18'$$

$\therefore$ $$2\theta = 2\times(81°18') = 162°36'$$

If n is the number of satellites needed to cover entire the earth, then

$$n = \frac{360°}{2\theta} = \frac{360°}{162°36'} = 2.31$$

$\therefore$ Minimum 3 satellites are required to cover entire the earth.