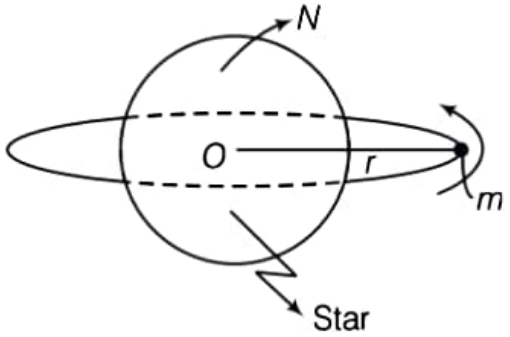
Q. 34 A star like the sun has several bodies moving around it at different distances. Consider that all of them are moving in circular orbits. Let r be the distance of the body from the centre of the star and let its linear velocity be v, angular velocity ω, kinetic energy K, gravitational potential energy U, total energy E and angular momentum l. As the radius r of the orbit increases, determine which of the above quantities increase and which ones decrease.

Ans. The situation is shown in the diagram, where a body of mass m is revolving around a star of mass M.



Linear velocity of the body $\quad v = \sqrt{\dfrac{GM}{r}}$

$\Rightarrow \quad v \propto \dfrac{1}{\sqrt{r}}$

Therefore, when r increases, v decreases.

Angular velocity of the body $\quad \omega = \dfrac{2\pi}{T}$

According to Kepler's law of period,

$$T^2 \propto r^3 \Rightarrow T = kr^{3/2}$$

where k is a constant

$$\therefore \quad \omega = \frac{2\pi}{kr^{3/2}} \Rightarrow \omega \propto \frac{1}{r^{3/2}} \qquad \left(\because \omega = \frac{2\pi}{T}\right)$$

Therefore, when r increases, ω decreases.

Kinetic energy of the body $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}m \times \dfrac{GM}{r} = \dfrac{GMm}{2r}$

$$\therefore \quad K \propto \frac{1}{r}$$

Therefore, when r increases, KE decreases.

Gravitational potential energy of the body,

$$U = -\frac{GMm}{r} \Rightarrow U \propto -\frac{1}{r}$$

Therefore, when r increases, PE becomes less negative *i.e.*, increases.

Total energy of the body $\quad E = \text{KE} + \text{PE} = \dfrac{GMm}{2r} + \left(-\dfrac{GMm}{r}\right) = -\dfrac{GMm}{2r}$

Therefore, when r increases, total energy becomes less negative, *i.e.*, increases.

Angular momentum of the body $L = mvr = mr\sqrt{\dfrac{GM}{r}} = m\sqrt{GMr}$

$$\therefore \quad L \propto \sqrt{r}$$

Therefore, when r increases, angular momentum L increases.