

Problem 1 : A committee of 7 members is to be chosen from 6 artists, 4 singers, and 5 writers. In how many ways can this be done if in the committee there must be at least one member from each group and at least 3 artists?

Solution : For the given condition, possible ways to select members for a committee of 7 members.

$$(3A, 3S, 1W) \text{ ----} \rightarrow 6C3 \cdot 4C3 \cdot 5C1 = 20 \cdot 4 \cdot 5 = 400$$

$$(3A, 1S, 3W) \text{ ----} \rightarrow 6C3 \cdot 4C1 \cdot 5C3 = 20 \cdot 4 \cdot 10 = 800$$

$$(3A, 2S, 2W) \text{ ----} \rightarrow 6C3 \cdot 4C2 \cdot 5C2 = 20 \cdot 6 \cdot 10 = 1200$$

$$(4A, 2S, 1W) \text{ ----} \rightarrow 6C4 \cdot 4C2 \cdot 5C1 = 15 \cdot 6 \cdot 5 = 450$$

$$(4A, 1S, 2W) \text{ ----} \rightarrow 6C4 \cdot 4C1 \cdot 5C2 = 15 \cdot 4 \cdot 10 = 600$$

$$(5A, 1S, 1W) \text{ ----} \rightarrow 6C5 \cdot 4C1 \cdot 5C1 = 6 \cdot 4 \cdot 5 = 120$$

Thus, the total no. of ways is

$$= 400 + 800 + 1200 + 450 + 600 + 120$$

$$= 3570$$