Question

A double star is a system of two stars of masses m and $2m$, rotating about their centre of mass only under their mutual gravitational attraction. If r is the separation between these two stars then their time period of rotation about their centre of mass will be proportional to:

This question has multiple correct options

A $r^{\frac{3}{2}}$

B r

C $m^{\frac{1}{2}}$

D $m^{-\frac{1}{2}}$

Solution

Correct options are A) and D)

For equilibrium, $F_{gravitational} = F_{centrifugal}$

Distance between the stars is r. So

$$F_{gravitational} = \frac{Gm(2m)}{r^2}$$

For point O to be the center of mass of the system, then $3m(x) = 0 + 2m(r) \Rightarrow x = \frac{2r}{3}$

Let the star of mass m revolves around O with angular velocity w.

$F_{centrifugal} = m(\frac{2r}{3})w^2$; where, $w = \frac{2\pi}{T}$

Thus, $\frac{Gm(2m)}{r^2} = m(\frac{2r}{3})\frac{4\pi^2}{T^2}$

$$\Rightarrow T^2 = \frac{4\pi^2 r^3}{3Gm}$$

Hence, $T \propto r^{\frac{3}{2}} m^{\frac{-1}{2}}$