Question

A very long (length L) cylindrical galaxy is made of uniformly distributed mass and has radius R(R << L). A star outside the galaxy is orbiting the galaxy in a plane perpendicular to the galaxy and passing through its centre. If the time period of star is t and its distance from the galaxy's axis is r, then

A $T \propto r^2$

B $T \propto r$

C $T \propto \sqrt{r}$

D $T^2 \propto r^3$

Solution

Correct option is B)

The gravitational potential due to cylindrical mass distribution can be found by comparison with electric field due to cylindrical charge distribution.

Applying gauss law for cylindrical charge distribution, consider a cylindrical Gaussian surface of length L:

$$\oint \vec{E}.d\vec{A} = \frac{Q_{en}}{\epsilon}$$

$$E 2\pi r L = \frac{\lambda L}{\epsilon}$$

$$E = \frac{\lambda}{2\pi\epsilon r}$$

Similarly gravitational field =

$E = \frac{2G\lambda}{r}$ where $\frac{1}{4\pi\epsilon} = G$ from similarity

As the Gravitational field is inversely proportional to distance from center, The centripetal force is

$$F_{centripetal} = mE = m\frac{2G\lambda}{r} \text{ ...(i)}$$

$$F_{centripetal} = m\omega^2 r\text{...(ii)}$$

substituting value of centripetal force in equation (i) from equation (ii)

$$m\omega^2 r = m\frac{2G\lambda}{r}$$

$$\omega = \frac{1}{r}\sqrt{2G\lambda}$$

$T = \frac{2\pi}{\omega} = \frac{r}{\sqrt{2G\lambda}} \Rightarrow T \alpha r$ hence correct answer is option B