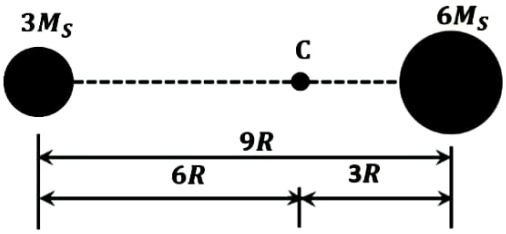
The distance between two stars of masses $3M_S$ and $6M_S$ is 9R. Here R is the mean distance between the centers of the Earth and the Sun, and M_S is the mass of the Sun. The two stars orbit around their common center of mass in circular orbits with period nT, where T is the period of Earth's revolution around the Sun. The value of n is _____.

Solution

The time period of earth revolving around the sun is given by

$$T = 2\pi\sqrt{\frac{R^3}{GM_S}}$$



Centre of mass of two stars lies at x from $3M_S$, which is

$$x = \frac{(3M_S \times 0) + (6M_S \times 9R)}{(3M_S + 6M_S)} = 6R$$

Let T' be the time period of star having mass $3M_S$ and v be its velocity.

$$\frac{3M_S v^2}{6R} = \frac{G(3M_S)(6M_S)}{(9R)^2}$$

$$v = \frac{2}{3}\sqrt{\frac{GM_S}{R}}$$

$$\therefore T' = \frac{2\pi(6R)}{v} = \frac{18\pi R\sqrt{R}}{\sqrt{GM_S}} = 9 \times 2\pi\sqrt{\frac{R^3}{GM_S}}$$

$$\Rightarrow T' = 9T$$

$$\therefore n = 9$$