Question

Distance between the centres of two stars is 10a. The masses of these stars are M and 16M and their radii a and 2a respectively. A body of mass m is fired straight from the surface of the larger star towards the smaller star. What should be its minimum initial speed to reach the surface of the smaller star? Obtain the expression in terms of G, M and a.

Solution

Let there are two stars 1 and 2 as shown in figure

Let P is a point between C_1 and C_2, where gravitational field strength is zero. Or at P field strength due to star 1 is equal and opposite to the field strength due to star 2. Hence,

$$\frac{GM}{r_1^2} = \frac{G(16M)}{r_2^2}$$

$$\text{or } \frac{r_2}{r_1} = 4$$

also $r_1 + r_2 = 10a$

$$\therefore\ r_2 = \left(\frac{4}{4+1}\right)(10a) = 8a$$

and $r_1 = 2a$

Now, the body of mass m is projected from the surface of larger star towards the smaller one. Between C_2 and P it is attracted towards 2 and between C_1 and P it will be attracted towards 1. Therefore, the body should be projected to just cross point P because beyond that the particle attracted towards the smaller star itself. From conservation of mechanical energy

$\frac{1}{2}mv_{min}^2$

=Potential energy of the body at P- Potential energy at the surface of the larger star.

$$\therefore\ \frac{1}{2}mv_{min}^2 = \left[-\frac{GMm}{r_1} - \frac{16GMm}{r_2}\right] - \left[-\frac{GMm}{10a-2a} - \frac{16GMm}{2a}\right]$$

$$= \left[-\frac{GMm}{2a} - \frac{16GMm}{8a}\right] - \left[-\frac{GMm}{8a} - \frac{8GMm}{a}\right]$$

$$\text{or } \frac{1}{2}mv_{min}^2 = \left(\frac{45}{8}\right)\frac{GMm}{a}$$

$$\therefore\ v_{min} = \frac{3\sqrt{5}}{2}\left(\sqrt{\frac{GM}{a}}\right)$$