Let a, b and c be in G.P with common ratio r, where $a \neq 0$ and $0 < r \leq \frac{1}{2}$. If $3a, 7b$ and $15c$ are the first three terms of an A.P, then the 4th term of this A.P is:

(JEE Main 2019)

Solution:

$b = ar, \quad c = ar^2$

$\because$ $3a, 7b$ and $15c$ are in AP

$\Rightarrow 14b = 3a + 15c$

$\Rightarrow 14ar = 3a + 15ar^2$

$\Rightarrow 14r = 3 + 15r^2$

$\Rightarrow 15r^2 - 14r + 3 = 0$

$\Rightarrow (3r-1)(5r-3) = 0$

$r = 1/3, 3/5$

Only acceptable value is $r = \frac{1}{3}$ because

$r \in \left(0, \frac{1}{2}\right]$

$\therefore$ common difference $= 7b - 3a = 7ar - 3a = -\frac{2}{3}a$

$\therefore$ 4th term $= 15c - \frac{2}{3}a = \frac{15}{9}a - \frac{2}{3}a = a$