Let α and β be roots of equation $px^2+qx+r=0$, $p\neq 0$. If p, q, r are in AP and $\frac{1}{\alpha}+\frac{1}{\beta}=4$, then the value of $|\alpha-\beta|$ is

(JEE Main 2014)

Solution:

$\because \frac{1}{\alpha}+\frac{1}{\beta}=4 \Rightarrow \frac{\alpha+\beta}{\alpha\beta}=4=-\frac{q}{r}$

(Sum and product of roots)

$\Rightarrow \boxed{q=-4r}$

$\because p, q, r$ are in AP

$\Rightarrow 2q=p+r \Rightarrow -8r=p+r$

$\Rightarrow \boxed{p=-9r}$

$\therefore |\alpha-\beta|=\sqrt{(\alpha+\beta)^2-4\alpha\beta}=\sqrt{\frac{16r^2+36r^2}{p^2}}$

$=\sqrt{\frac{52r^2}{p^2}}=\sqrt{\frac{52}{81}}=\frac{2\sqrt{13}}{9}$