

Related problem :

A couple has two children,

- (i) Find the probability that both children are males, if it is known that at least one of the children is male.
- (ii) Find the probability that both children are females, if it is known that the elder child is a female.

Solution:

If a couple has two children, then the sample space is $S = \{(B, B), (B, G), (G, B), (G, G)\}$

- (i) Let E and F respectively denote the events that both children are male and atleast one children is a male.

$$E \cap F = \{(G, G)\} \Rightarrow P(E \cap F) = \frac{1}{4}$$

$$P(E) = \frac{1}{4}$$

$$P(F) = \frac{3}{4}$$

$$\Rightarrow P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$= \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

- (ii) Let C and D respectively denote the events that both children are females and the elder child is a female.

$$C = \{(G, G)\} \Rightarrow P(C) = \frac{1}{4}$$

$$D = \{(G, B), (G, G)\} \Rightarrow P(D) = \frac{2}{4}$$

$$C \cap D = \{(G, G)\} \Rightarrow P(C \cap D) = \frac{1}{4}$$

$$\therefore P(C|D) = \frac{P(C \cap D)}{P(D)} = \frac{\frac{1}{4}}{\frac{2}{4}} = \frac{1}{2}$$