

## Related Problem with Solution

Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die?

**Solution:**

Let  $E_1$  be the event that the outcome on the die is 5 or 6 and  $E_2$  be the event that the outcome on the die is 1, 2, 3, or 4.

$$P(E_1) = \frac{2}{6} = \frac{1}{3} \text{ and } P(E_2) = \frac{4}{6} = \frac{2}{3}$$

Let A be the event of getting exactly one head.

Probability of getting exactly one head by tossing the coin three times if she gets 5 or 6,

$$P(A|E_1) = \frac{3}{8}$$

Probability of getting exactly one head in a single throw of coin if she gets 1, 2, 3, or 4,

$$P(A|E_2) = \frac{1}{2}$$

The probability that the girl threw 1, 2, 3, or 4 with die, if she obtained exactly one head, is given by  $P(E_2|A)$ .

By using Bayes' theorem, we get

$$\begin{aligned} P(E_2|A) &= \frac{P(E_2) \cdot P(A|E_2)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)} \\ &= \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{1}{2}} \\ &= \frac{\frac{1}{3}}{\frac{1}{3} + 1} \\ &= \frac{1}{\frac{3}{3} + 1} \\ &= \frac{1}{1 + 1} = \frac{1}{2} \end{aligned}$$