Related Problem with Solution

A manufacture has three machine operators A, B and C. The first operator A produces 1% defective items, whereas the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that was produced by A?

Solution:

Let E_1, E_2 and E_3 be the respective events that the time consumed by machine A, B, and C for the job.

$$P(E_1) = 50\% = \frac{50}{100} = \frac{1}{2}$$

$$P(E_2) = 30\% = \frac{30}{100} = \frac{3}{10}$$

$$P(E_3) = 20\% = \frac{20}{100} = \frac{1}{5}$$

Let X be the event of producing defective items.

$$P(X|E_1) = 1\% = \frac{1}{100}$$

$$P(X|E_2) = 5\% = \frac{5}{100}$$

$$P(X|E_3) = 7\% = \frac{7}{100}$$

The probability that the defective item was produced by A is given by $P(E_1|X)$.

By using Bayes' theorem, we get

$$P(E_1|X) = \frac{P(E_1)\cdot P(X|E_1)}{P(E_1)\cdot P(X|E_1) + P(E_2)\cdot P(X|E_2) + P(E_3)\cdot P(X|E_3)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{100}}{\frac{1}{2}\times\frac{1}{100} + \frac{3}{10}\times\frac{5}{100} + \frac{1}{5}\times\frac{7}{100}}$$

$$= \frac{\frac{1}{2}\times\frac{1}{100}}{\frac{1}{100}\left(\frac{1}{2} + \frac{3}{2} + \frac{7}{5}\right)}$$

$$= \frac{\frac{1}{2}}{\frac{17}{5}} = \frac{5}{34}$$