Q.27 The entropy of any system is given by

$S = \alpha^2\beta \ln\left[\frac{\mu kR}{J\beta^2} + 3\right]$ where α and β are the constants. μ, J, k and R are no. of moles, mechanical equivalent of heat, Boltzmann constant and gas constant respectively.

[Take $S = \frac{dQ}{T}$]

Choose the incorrect option from the following :

(A) α and J have the same dimensions.

(B) S and α have different dimensions

(C) S, β, k and μR have the same dimensions

(D) α and k have the same dimensions

20th July Morning Shift 2021

Ans. D

Ans 27. Since, entropy of the system is given by

$$S = \alpha^2 \beta \ln\left[\frac{\mu k R}{J\beta^2} + 3\right] \ldots\ldots (i)$$

As, $S = \frac{Q}{\Delta T}$ [given]

$$\Rightarrow [S] = \frac{[ML^2T^{-2}]}{[K]} \ldots\ldots (ii)$$

∵ Dimensions of Q = $[ML^2T^{-2}]$

Dimension of T = [K]

Boltzmann constant, $k = \frac{energy}{T}$ [∵ Dimensions of energy = $[ML^2T^{-2}]$]

$$\Rightarrow [k] = \frac{[ML^2T^{-2}]}{[K]} \ldots\ldots (iii)$$

From Eqs. (ii) and (iii), we can write,

$$[S] = [k] = \frac{[ML^2T^{-2}]}{[K]} \ldots\ldots (iv)$$

∵ Gas constant, $[R] = \frac{[Energy]}{[nT]} = \frac{[ML^2T^{-2}]}{[mol\ K]} \ldots\ldots (v)$

and mechanical equivalent of heat

$[J] = [M^0L^0T^0] \ldots\ldots (vi)$

As, $[\mu kR] = [J\beta]^2$

Using Eqs. (iii), (v) and (vi), we get

$$\Rightarrow [mol] \times \frac{[ML^2T^{-2}]}{[K]} \times \frac{[ML^2T^{-2}]}{[mol\ K]} = [\beta^2]$$

$$\Rightarrow [\beta] = [ML^2T^{-2}K^{-1}] \ldots\ldots (vii)$$

Using Eq. (i), we can write,

$$[\alpha^2] = \frac{[S]}{[\beta]} = \frac{[ML^2T^{-2}K^{-1}]}{[ML^2T^{-2}K^{-1}]} \Rightarrow \alpha = [M^0L^0T^0] \ldots\ldots (viii)$$

So, from Eqs. (iii) and (viii), we can say that α and k have different dimensions.