17. Which of the following is a homogeneous differential equation?

- (A) $(4x + 6y + 5) dy (3y + 2x + 4) dx = 0$
- (B) (*xy*) $dx (x^3 + y^3) dy = 0$
- (C) $(x^3 + 2y^2) dx + 2xy dy = 0$
- (D) $y^2 dx + (x^2 xy y^2) dy = 0$

9.5.3 *Linear differential equations*

A differential equation of the from

$$
\frac{dy}{dx} + Py = Q
$$

where, P and Q are constants or functions of *x* only, is known as a first order linear differential equation. Some examples of the first order linear differential equation are

(A)
$$
(4x + 6y + 3) dy - (3y + 2x + 4) dx = 0
$$

\n(B) $(xy) dx - (x^2 + y^2) dy = 0$
\n(C) $(x^3 + 2y^2) dx + (x^2 - xy - y^2) dy = 0$
\n9.5.3 Linear differential equations
\nA differential equation of the from
\n
$$
\frac{dy}{dx} + Py = Q
$$
\nwhere, P and Q are constants or functions of x only, is known as a first order linear
\ndifferential equation. Some examples of the first order linear differential equation are
\n
$$
\frac{dy}{dx} + y = \sin x
$$
\n
$$
\frac{dy}{dx} + (\frac{y}{x})y = e^x
$$
\n
$$
\frac{dy}{dx} + (\frac{y}{x \log x}) = \frac{1}{x}
$$
\nAnother form of first order linear differential equation is
\n
$$
\frac{dx}{dy} + P_xx = Q_1
$$
\nwhere, P, and Q₁ are constants or functions of y only. Some examples of this type of
\ndifferential equation are
\n
$$
\frac{dx}{dy} + x = \cos y
$$
\n
$$
\frac{dx}{dy} + x = \cos y
$$
\n
$$
\frac{dx}{dy} + \frac{-2x}{y} = y^2e^{-y}
$$
\nTo solve the first order linear differential equation of the type
\n
$$
\frac{dy}{dx} + P_x = Q_1
$$
\n
$$
\frac{dx}{dy} + x = \cos y
$$
\n
$$
\frac{dx}{dy} + \frac{-2x}{y} = y^2e^{-y}
$$
\nTo solve the first order linear differential equation of the type
\n
$$
\frac{dy}{dx} + P_x(g(x)) = Q_x(g(x))
$$
\n...(1)

Another form of first order linear differential equation is

$$
\frac{dx}{dy} + P_1 x = Q_1
$$

where, P_1 and Q_1 are constants or functions of *y* only. Some examples of this type of differential equation are

$$
\frac{dx}{dy} + x = \cos y
$$

$$
\frac{dx}{dy} + \frac{-2x}{y} = y^2 e^{-y}
$$

To solve the first order linear differential equation of the type

$$
\frac{dy}{dx} \quad Py = Q \qquad \qquad \dots (1)
$$

Multiply both sides of the equation by a function of *x* say $g(x)$ to get

$$
g(x) \frac{dy}{dx} + P.(g(x)) y = Q.g(x)
$$
 ... (2)

408 MATHEMATICS

Choose $g(x)$ in such a way that R.H.S. becomes a derivative of $y \cdot g(x)$.

i.e.
$$
g(x) \frac{dy}{dx} + P. g(x) y = \frac{d}{dx} [y. g(x)]
$$

or $g(x)$

$$
\frac{dy}{dx} + P. g(x) y = g(x) \frac{dy}{dx} + y g'(x)
$$

$$
\Rightarrow \qquad \qquad \mathsf{P}. \, g\left(x\right) = g'\left(x\right)
$$

or $P = \frac{g'(x)}{g(x)}$ *g x g x* ′

Integrating both sides with respect to *, we get*

$$
\int Pdx = \int \frac{g'(x)}{g(x)} dx
$$

or
$$
\int P \cdot dx = \log(g(x))
$$

or $g(x) = e^{\int P dx}$

On multiplying the equation (1) by $g(x) = e^{\int P dx}$, the L.H.S. becomes the derivative of some function of *x* and *y*. This function $g(x) = e^{\int P dx}$ is called *Integrating Factor* (I.F.) of the given differential equation. or $\int P \cdot dx =$

or $g(x) = g(x) =$

On multiplying the equation (1) by

of some function of x and y. This function

(I.F.) of the given differential equation

Substituting the value of g (x) in equation
 $e^{P dx} \frac{dy}{dx} = e^{P dx} \frac{dy}{$ i.e. $g(x) \frac{dy}{dx} + P_x g(x) y = \frac{d}{dx} [y \cdot g(x)]$

or $g(x) \frac{dy}{dx} + P_x g(x) y = g(x) \frac{dy}{dx} + y g'(x)$
 $\Rightarrow P_x g(x) = g'(x)$
 $\Rightarrow P_y g(x) = g'(x)$

Integrating both sides with respect to x , we get
 $\int P dx = \int \frac{g'(x)}{g(x)} dx$

or $\int P dx = \ln(g(g(x)))$

or $g(x) = e^{\int p_x$

Substituting the value of $g(x)$ in equation (2), we get

$$
e^{P dx} \frac{dy}{dx} P e^{P dx} y = Q e^{P dx}
$$

$$
\frac{d}{dx} y e^{P dx} = Q e^{P dx}
$$

or

Integrating both sides with respect to x , we get

$$
y e^{Pdx} = Q e^{Pdx} dx
$$

or

$$
y = e^{Pdx} Q e^{Pdx} dx
$$

which is the general solution of the differential equation.