EXERCISE 9.5

In each of the Exercises 1 to 10, show that the given differential equation is homogeneous and solve each of them.

1. $(x^2 + xy)\, dy = (x^2 + y^2)\, dx$

2. $y' = \dfrac{x+y}{x}$

3. $(x - y)\, dy - (x + y)\, dx = 0$

4. $(x^2 - y^2)\, dx + 2xy\, dy = 0$

5. $x^2 \dfrac{dy}{dx} = x^2 - 2y^2 + xy$

6. $x\, dy - y\, dx = \sqrt{x^2 + y^2}\, dx$

7. $\left\{x\cos\left(\dfrac{y}{x}\right) + y\sin\left(\dfrac{y}{x}\right)\right\} y\, dx = \left\{y\sin\left(\dfrac{y}{x}\right) - x\cos\left(\dfrac{y}{x}\right)\right\} x\, dy$

8. $x\dfrac{dy}{dx} - y + x\sin\left(\dfrac{y}{x}\right) = 0$

9. $y\, dx + x\log\left(\dfrac{y}{x}\right)dy - 2x\, dy = 0$

10. $1 \quad e^{\frac{x}{y}} \quad dx \quad e^{\frac{x}{y}} \quad 1 \quad \dfrac{x}{y} \quad dy \quad 0$

For each of the differential equations in Exercises from 11 to 15, find the particular solution satisfying the given condition:

11. $(x + y)\, dy + (x - y)\, dx = 0$; $y = 1$ when $x = 1$

12. $x^2\, dy + (xy + y^2)\, dx = 0$; $y = 1$ when $x = 1$

13. $x\sin^2 \dfrac{y}{x} \quad y \quad dx \quad x\, dy \quad 0;\ y \quad \dfrac{}{4}$ when $x = 1$

14. $\dfrac{dy}{dx} - \dfrac{y}{x} + \text{cosec}\left(\dfrac{y}{x}\right) = 0$; $y = 0$ when $x = 1$

15. $2xy + y^2 - 2x^2\dfrac{dy}{dx} = 0$; $y = 2$ when $x = 1$

16. A homogeneous differential equation of the from $\dfrac{dx}{dy} = h\left(\dfrac{x}{y}\right)$ can be solved by making the substitution.

(A) $y = vx$ (B) $v = yx$ (C) $x = vy$ (D) $x = v$

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17. Which of the following is a homogeneous differential equation?

(A) $(4x + 6y + 5)\, dy - (3y + 2x + 4)\, dx = 0$

(B) $(xy)\, dx - (x^3 + y^3)\, dy = 0$

(C) $(x^3 + 2y^2)\, dx + 2xy\, dy = 0$

(D) $y^2\, dx + (x^2 - xy - y^2)\, dy = 0$

9.5.3 *Linear differential equations*

A differential equation of the from

$$\frac{dy}{dx} + Py = Q$$

where, P and Q are constants or functions of x only, is known as a first order linear differential equation. Some examples of the first order linear differential equation are

$$\frac{dy}{dx} + y = \sin x$$

$$\frac{dy}{dx} + \left(\frac{1}{x}\right) y = e^x$$

$$\frac{dy}{dx} + \left(\frac{y}{x \log x}\right) = \frac{1}{x}$$

Another form of first order linear differential equation is

$$\frac{dx}{dy} + P_1 x = Q_1$$

where, P_1 and Q_1 are constants or functions of y only. Some examples of this type of differential equation are

$$\frac{dx}{dy} + x = \cos y$$

$$\frac{dx}{dy} + \frac{-2x}{y} = y^2 e^{-y}$$

To solve the first order linear differential equation of the type

$$\frac{dy}{dx} \quad Py = Q \qquad \text{... (1)}$$

Multiply both sides of the equation by a function of x say $g(x)$ to get

$$g(x) \frac{dy}{dx} + P.(g(x))\, y = Q \,.\, g(x) \qquad \text{... (2)}$$

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