or 
$$
\sin v = \log |x| + \log |C|
$$
  
or 
$$
\sin v = \log |Cx|
$$

Replacing *v* by  $\frac{y}{x}$ , we get

$$
\sin\left(\frac{y}{x}\right) = \log|Cx|
$$

which is the general solution of the differential equation (1).

**Example 17** Show that the differential equation  $2y e^{y} dx + (y-2x e^{y}) dy = 0$ *x x*  $dv = 0$  is homogeneous and find its particular solution, given that,  $x = 0$  when  $y = 1$ . **Solution** The given differential equation can be written as or<br>
Replacing v by  $\frac{y}{x}$ , we get<br>  $\sin(\frac{y}{x}) = \log |\text{Cx}|$ <br>
which is the general solution of the differential equation (1).<br>
Example 17 Show that the differential equation (1).<br>
Example 17 Show that the differential equati

$$
\frac{dx}{dy} = \frac{2xe^{\frac{x}{y}} - y}{2ye^{\frac{x}{y}}}
$$
 ... (1)

Let 
$$
F(x, y) = \frac{2xe^{\frac{x}{y}} - y}{2ye^{\frac{x}{y}}}
$$

Then 
$$
F(\lambda x, \lambda y) = \frac{\lambda \left(2xe^{\frac{x}{y}} - y\right)}{\lambda \left(2ye^{\frac{x}{y}}\right)} = \lambda^{0}[F(x, y)]
$$

Thus,  $F(x, y)$  is a homogeneous function of degree zero. Therefore, the given differential equation is a homogeneous differential equation. To solve it, we make the substitution Let  $F(x, y) =$ <br>
Then  $F(\lambda x, \lambda y) =$ <br>
Thus,  $F(x, y)$  is a homogeneous<br>
differential equation is a homogeneous<br>
To solve it, we make the substitution  $x =$ <br>
Differentiating equation (2) with respectively

$$
x = vy \tag{2}
$$

Differentiating equation (2) with respect to *y*, we get

$$
\frac{dx}{dy} = v + y\frac{dv}{dy}
$$

## 404 MATHEMATICS

Substituting the value of x and  $\frac{dx}{dy}$  in equation (1), we get

$$
v + y\frac{dv}{dy} = \frac{2v e^v - 1}{2e^v}
$$

 $y\frac{dv}{dy} = \frac{2v e^{v} - 1}{2e^{v}}$ 

*v v*  $\frac{v e^v - 1}{v} - v$ *e*

 $\frac{-1}{-}$ 

or

or 
$$
y \frac{dv}{dy} = -\frac{1}{2e^v}
$$

$$
2e^{\nu} dv = \frac{-dy}{y}
$$

or  $\int 2e^v \cdot dv = -\int \frac{dy}{y}$ 

or  $2 e^{v} = -\log |y| + C$ 

and replacing *v* by *x*  $\frac{1}{y}$ , we get

$$
2e^{y} + \log|y| = C \qquad \dots (3)
$$

Substituting  $x = 0$  and  $y = 1$  in equation (3), we get

*x*

$$
e^0 + \log |1| = C \Rightarrow C = 2
$$

Substituting the value of C in equation (3), we get

 $\overline{2}$ 

2

$$
\frac{x}{e^y} + \log|y| = 2
$$

which is the particular solution of the given differential equation.

**Example 18** Show that the family of curves for which the slope of the tangent at any

point  $(x, y)$  on it is  $\frac{x^2 + y^2}{ }$ 2  $x^2 + y$ *xy*  $+\frac{y^2}{x}$ , is given by  $x^2 - y^2 = cx$ . or  $2e^y =$ <br>
and replacing  $v$  by  $\frac{x}{y}$ , we get<br>  $2e^y + \log |y| =$ <br>
Substituting  $x = 0$  and  $y = 1$  in equatio<br>  $2e^0 + \log |1| =$ <br>
Substituting the value of C in equation<br>  $2e^y + \log |y| =$ <br>
which is the particular solution of the g ontinuiting the standard  $\frac{dy}{dx} = \frac{2y e^x - 1}{2e^x}$ <br>
or  $y \frac{dv}{dy} = \frac{2v e^x - 1}{2e^x} - v$ <br>
or  $y \frac{dv}{dy} = -\frac{1}{2e^x}$ <br>
or  $2e^x dv = -\frac{dy}{y}$ <br>
or  $2e^x dv = \frac{-dy}{y}$ <br>
or  $\left[2e^x \cdot dv - \frac{1}{2} \frac{dy}{y}\right]$ <br>
or  $2e^x - \log |v| + C$ <br>
and replaci

**Solution** We know that the slope of the tangent at any point on a curve is  $\frac{dy}{dx}$ .

Therefore, *dy*  $\frac{dy}{dx}$  =  $2^{1}$   $2^{2}$ 2  $x^2 + y$ *xy* +

or 
$$
\frac{dy}{dx} = \frac{1 + \frac{y^2}{x^2}}{\frac{2y}{x}}
$$
 ... (1)  
\nClearly, (1) is a homogeneous differential equation. To solve it we make substitution  
\n $y = vx$   
\nDifferentiating  $y = vx$  with respect to x, we get  
\n
$$
\frac{dy}{dx} = v + x\frac{dv}{dx}
$$
\nor  
\n
$$
v + x\frac{dv}{dx} = \frac{1 + v^2}{2v}
$$
\nor  
\n
$$
x\frac{dv}{dx} = \frac{1 - v^2}{2v}
$$
\nor  
\n
$$
\frac{2v}{v^2 - 1}dv = \frac{dx}{x}
$$
\n
$$
\frac{2v}{v^2 - 1}dv = -\int \frac{1}{x}dx
$$
\nor  
\n
$$
\log |v^2 - 1| = -\log |x| + \log |C_1|
$$
\nor  
\n
$$
\log |v^2 - 1| = -\log |x| + \log |C_1|
$$
\nor  
\n
$$
(v^2 - 1) (x) = \log |C_1|
$$
\nor  
\n
$$
(v^2 - 1) (x) = \log |C_1|
$$
\nor  
\n
$$
(y^2 - x^2) = \pm C_1 x
$$
 or  $x^2 - y^2 = Cx$ 

or

Clearly, (1) is a homogenous differential equation. To solve it we make substitution

$$
y = \nu x
$$

Differentiating  $y = vx$  with respect to *x*, we get

## $\frac{dy}{dx} = v + x \frac{dv}{dx}$ *dx* +

2 *v v* +

 $\frac{dx}{x}$ 

or

or

$$
x\frac{dv}{dx} = \frac{1 - v^2}{2v}
$$

 $v + x \frac{dv}{dx}$ *dx*  $+x\frac{dv}{dt}=\frac{1+v^2}{2}$ 

$$
\frac{2v}{1-v^2}dv = \frac{dx}{x}
$$

 $\frac{2v}{v^2-1}dv = -\frac{dx}{x}$ 1

or  $\frac{1}{x^2}$ 

Therefore 
$$
\int \frac{2v}{v^2 - 1} dv = -\int \frac{1}{x} dx
$$

or  
\n
$$
x \frac{dv}{dx} = \frac{1 - v^2}{2v}
$$
\n
$$
\frac{2v}{1 - v^2} dv = \frac{dx}{x}
$$
\nor  
\n
$$
\frac{2v}{v^2 - 1} dv = -\frac{dx}{x}
$$
\nTherefore  
\n
$$
\int \frac{2v}{v^2 - 1} dv = -\int \frac{1}{x} dx
$$
\nor  
\n
$$
\log |v^2 - 1| = -\log |x| + \log |C_1|
$$
\nor  
\n
$$
\log |(v^2 - 1)(x)| = \log |C_1|
$$
\nor  
\n
$$
(v^2 - 1) x = \pm C_1
$$
\nReplacing v by  $\frac{y}{x}$ , we get

Replacing *v* by  $\frac{y}{x}$ , we get

$$
\left(\frac{y^2}{x^2} - 1\right) x = \pm C_1
$$
  
or  

$$
(y^2 - x^2) = \pm C_1 x \text{ or } x^2 - y^2 = Cx
$$