Substituting $x = -2$, $y = 3$ in equation (3), we get C = 5.

Substituting the value of C in equation (3), we get the equation of the required curve as

$$\frac{y^3}{3} = x^2 + 5 \quad \text{or} \quad y = (3x^2 + 15)^{\frac{1}{3}}$$

Example 14 In a bank, principal increases continuously at the rate of 5% per year. In how many years Rs 1000 double itself?

Solution Let P be the principal at any time t. According to the given problem,

$$\frac{dp}{dt} = \left(\frac{5}{100}\right) \times \text{P}$$

or $$\frac{dp}{dt} = \frac{\text{P}}{20} \qquad \text{... (1)}$$

separating the variables in equation (1), we get

$$\frac{dp}{\text{P}} = \frac{dt}{20} \qquad \text{... (2)}$$

Integrating both sides of equation (2), we get

$$\log \text{P} = \frac{t}{20} + \text{C}_1$$

or $$\text{P} = e^{\frac{t}{20}} \cdot e^{\text{C}_1}$$

or $$\text{P} = \text{C}\, e^{\frac{t}{20}} \quad (\text{where } e^{\text{C}_1} = \text{C}) \qquad \text{... (3)}$$

Now $\text{P} = 1000$, when $t = 0$

Substituting the values of P and t in (3), we get C = 1000. Therefore, equation (3), gives

$$\text{P} = 1000\, e^{\frac{t}{20}}$$

Let t years be the time required to double the principal. Then

$$2000 = 1000\, e^{\frac{t}{20}} \quad \Rightarrow \quad t = 20 \log_e 2$$

EXERCISE 9.4

For each of the differential equations in Exercises 1 to 10, find the general solution:

1. $\dfrac{dy}{dx} = \dfrac{1 - \cos x}{1 + \cos x}$

2. $\dfrac{dy}{dx} = \sqrt{4 - y^2} \quad (-2 < y < 2)$

3. $\dfrac{dy}{dx} + y = 1 \; (y \neq 1)$

4. $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$

5. $(e^x + e^{-x}) \, dy - (e^x - e^{-x}) \, dx = 0$

6. $\dfrac{dy}{dx} = (1 + x^2)(1 + y^2)$

7. $y \log y \, dx - x \, dy = 0$

8. $x^5 \dfrac{dy}{dx} = -y^5$

9. $\dfrac{dy}{dx} = \sin^{-1} x$

10. $e^x \tan y \, dx + (1 - e^x) \sec^2 y \, dy = 0$

For each of the differential equations in Exercises 11 to 14, find a particular solution satisfying the given condition:

11. $(x^3 + x^2 + x + 1)\dfrac{dy}{dx} = 2x^2 + x$; $y = 1$ when $x = 0$

12. $x(x^2 - 1)\dfrac{dy}{dx} = 1$; $y = 0$ when $x = 2$

13. $\cos\left(\dfrac{dy}{dx}\right) = a \;\; (a \in \mathbf{R})$; $y = 2$ when $x = 0$

14. $\dfrac{dy}{dx} = y \tan x$; $y = 1$ when $x = 0$

15. Find the equation of a curve passing through the point (0, 0) and whose differential equation is $y' = e^x \sin x$.

16. For the differential equation $xy\dfrac{dy}{dx} = (x + 2)(y + 2)$, find the solution curve passing through the point (1, –1).

17. Find the equation of a curve passing through the point (0, –2) given that at any point (x, y) on the curve, the product of the slope of its tangent and y coordinate of the point is equal to the x coordinate of the point.

18. At any point (x, y) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point (– 4, –3). Find the equation of the curve given that it passes through (–2, 1).

19. The volume of spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of balloon after t seconds.

20. In a bank, principal increases continuously at the rate of r% per year. Find the value of r if Rs 100 double itself in 10 years ($\log_e 2 = 0.6931$).

21. In a bank, principal increases continuously at the rate of 5% per year. An amount of Rs 1000 is deposited with this bank, how much will it worth after 10 years ($e^{0.5} = 1.648$).

22. In a culture, the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000, if the rate of growth of bacteria is proportional to the number present?

23. The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is

(A) $e^x + e^{-y} = C$ (B) $e^x + e^y = C$

(C) $e^{-x} + e^y = C$ (D) $e^{-x} + e^{-y} = C$

9.5.2 *Homogeneous differential equations*

Consider the following functions in x and y

$F_1(x, y) = y^2 + 2xy$, $F_2(x, y) = 2x - 3y$,

$F_3(x, y) = \cos\left(\frac{y}{x}\right)$, $F_4(x, y) = \sin x + \cos y$

If we replace x and y by λx and λy respectively in the above functions, for any nonzero constant λ, we get

$$F_1(\lambda x, \lambda y) = \lambda^2 (y^2 + 2xy) = \lambda^2 F_1(x, y)$$

$$F_2(\lambda x, \lambda y) = \lambda (2x - 3y) = \lambda F_2(x, y)$$

$$F_3(\lambda x, \lambda y) = \cos\left(\frac{\lambda y}{\lambda x}\right) = \cos\left(\frac{y}{x}\right) = \lambda^0 F_3(x, y)$$

$$F_4(\lambda x, \lambda y) = \sin \lambda x + \cos \lambda y \neq \lambda^n F_4(x, y), \text{ for any } n \in \mathbf{N}$$

Here, we observe that the functions F_1, F_2, F_3 can be written in the form $F(\lambda x, \lambda y) = \lambda^n F(x, y)$ but F_4 can not be written in this form. This leads to the following definition:

A function $F(x, y)$ is said to be *homogeneous function of degree n* if

$F(\lambda x, \lambda y) = \lambda^n F(x, y)$ for any nonzero constant λ.

We note that in the above examples, F_1, F_2, F_3 are homogeneous functions of degree 2, 1, 0 respectively but F_4 is not a homogeneous function.