EXERCISE 9.3

In each of the Exercises 1 to 5, form a differential equation representing the given family of curves by eliminating arbitrary constants a and b.

1. $\dfrac{x}{a}+\dfrac{y}{b}=1$ **2.** $y^2 = a\,(b^2 - x^2)$ **3.** $y = a\,e^{3x} + b\,e^{-2x}$

4. $y = e^{2x}\,(a + bx)$ **5.** $y = e^x\,(a \cos x + b \sin x)$

6. Form the differential equation of the family of circles touching the y-axis at origin.

7. Form the differential equation of the family of parabolas having vertex at origin and axis along positive y-axis.

8. Form the differential equation of the family of ellipses having foci on y-axis and centre at origin.

9. Form the differential equation of the family of hyperbolas having foci on x-axis and centre at origin.

10. Form the differential equation of the family of circles having centre on y-axis and radius 3 units.

11. Which of the following differential equations has $y = c_1\,e^x + c_2\,e^{-x}$ as the general solution?

(A) $\dfrac{d^2y}{dx^2}+y=0$ (B) $\dfrac{d^2y}{dx^2}-y=0$ (C) $\dfrac{d^2y}{dx^2}+1=0$ (D) $\dfrac{d^2y}{dx^2}-1=0$

12. Which of the following differential equations has $y = x$ as one of its particular solution?

(A) $\dfrac{d^2y}{dx^2}-x^2\dfrac{dy}{dx}+xy=x$ (B) $\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+xy=x$

(C) $\dfrac{d^2y}{dx^2} \quad x^2\dfrac{dy}{dx} \quad xy \quad 0$ (D) $\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+xy=0$

9.5. Methods of Solving First Order, First Degree Differential Equations

In this section we shall discuss three methods of solving first order first degree differential equations.

9.5.1 *Differential equations with variables separable*

A first order-first degree differential equation is of the form

$$\frac{dy}{dx} = \mathrm{F}(x, y) \qquad \text{... (1)}$$

If F(x, y) can be expressed as a product g (x) $h(y)$, where, $g(x)$ is a function of x and $h(y)$ is a function of y, then the differential equation (1) is said to be of variable separable type. The differential equation (1) then has the form

$$\frac{dy}{dx} = h(y) \cdot g(x) \quad \text{... (2)}$$

If $h(y) \neq 0$, separating the variables, (2) can be rewritten as

$$\frac{1}{h(y)}\, dy = g(x)\, dx \quad \text{... (3)}$$

Integrating both sides of (3), we get

$$\int \frac{1}{h(y)}\, dy = \int g(x)\, dx \quad \text{... (4)}$$

Thus, (4) provides the solutions of given differential equation in the form

$$\mathrm{H}(y) = \mathrm{G}(x) + \mathrm{C}$$

Here, H (y) and G (x) are the anti derivatives of $\frac{1}{h(y)}$ and $g(x)$ respectively and C is the arbitrary constant.

Example 9 Find the general solution of the differential equation $\frac{dy}{dx} = \frac{x+1}{2-y}, (y \neq 2)$

Solution We have

$$\frac{dy}{dx} = \frac{x+1}{2-y} \quad \text{... (1)}$$

Separating the variables in equation (1), we get

$$(2 - y)\, dy = (x + 1)\, dx \quad \text{... (2)}$$

Integrating both sides of equation (2), we get

$$\int (2-y)\, dy = \int (x+1)\, dx$$

or $$2y - \frac{y^2}{2} = \frac{x^2}{2} + x + \mathrm{C}_1$$

or $$x^2 + y^2 + 2x - 4y + 2\,\mathrm{C}_1 = 0$$

or $$x^2 + y^2 + 2x - 4y + \mathrm{C} = 0, \text{ where } \mathrm{C} = 2\mathrm{C}_1$$

which is the general solution of equation (1).