

Question 2: Let $S(\alpha) = \{(x, y); y^2 \leq x, y^2 \leq \alpha\}$ and, $A(\alpha)$ is area of the region $S(\alpha)$. If for $\lambda, 0 < \lambda < 4$ and $A(\lambda) : A(4) = 2 : 5$. Then what will be the value of λ .

Solution:

Let $S(\alpha) = \{(x, y); y^2 \leq x, y^2 \leq \alpha\}$

$$A(\alpha) = 2 \int_0^\alpha \sqrt{x} \, dx = 2\alpha^{3/2}$$

$$A(4) = 2 \times 4^{3/2} = 16 \text{ and } A(\lambda) = 2 \times \lambda^{3/2}$$

As we know,

$$A(\lambda)/A(4) = 2/5$$

On putting values, we have

$$\Rightarrow \lambda = 4 (4/25)^{1/3}$$