

The equation  $e^{\sin x} - e^{-\sin x} - 4 = 0$  has :

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(1) exactly four real roots.

(2) infinite number of real roots.

(3) no real roots.

(4) exactly one real root.

Given  $e^{\sin x} - e^{-\sin x} = 4$ .

let  $e^{\sin x} = y$

$$y - \frac{1}{y} = 4 \Rightarrow y^2 - 4y - 1 = 0$$

$$y = 2 \pm \sqrt{5}$$
$$e^{\sin x} = 2 + \sqrt{5} \qquad e^{\sin x} = 2 - \sqrt{5}$$

but we know that

$$e^{-1} \leq e^{\sin x} \leq e^1$$

so  $e^{\sin x} \neq 2 + \sqrt{5}$  and  $2 - \sqrt{5}$

so No real solution of given equation.