

$$\int x \sin^{-1} x \, dx$$

$$I = \int x \sin^{-1} x \, dx$$

$$\text{Let, } x = \sin \theta$$

$$dx = \cos \theta \, d\theta$$

$$I = \int \theta \sin \theta \, d\theta$$

Using by parts

$$I = \theta \int \sin \theta \, d\theta - \int \frac{d\theta}{d\theta} \int \sin \theta \, d\theta \cdot d\theta + c$$

$$I = -\theta \cos \theta + \sin \theta + c$$

$$\text{Put } \theta = \sin^{-1} x$$

$$I = \sin(\sin^{-1} x) - \cos(\sin^{-1} x) \cdot \sin^{-1} x + c$$