(9) $\int e^{-3x} \sin 4x \, dx$

$I = \int e^{-3x} \sin 4x \, dx$

Apply by parts here

$I = e^{-3x} \cdot \int \sin 4x - \int \left(\frac{de^{-3x}}{dx}\right) \cdot \left(\int \sin 4x \, dx\right) dx + C$

$I = -\frac{e^{-3x} \cdot \cos 4x}{4} - \frac{3}{4}\int e^{-3x} \cos 4x \, dx + C$

Apply by parts again

$I = -\frac{e^{-3x} \cos 4x}{4} - \frac{3}{16} e^{-3x} \frac{\sin 4x}{4} - \frac{3}{4}\int e^{-3x} \sin 4x \, dx$

$+ C$

It is I itself

$$\therefore \quad I = \frac{-e^{-3x}\cos 4x}{4} - \frac{3}{16}e^{-3x}\sin 4x - \frac{3}{4}I + c$$

$$\frac{I}{4} = \frac{-e^{-3x}}{4}\left(\cos 4x + \frac{3\sin 4x}{4}\right) + c$$

$$I = -e^{-3x}\left(\cos 4x + \frac{3\sin 4x}{4}\right) + c$$