Q) $\int \left(1+x-\frac{1}{x}\right) e^{x+\frac{1}{x}} dx$ is equal to

[JEE Main 2014]

$\rightarrow$ let,

$$I = \int \left(1+x-\frac{1}{x}\right) e^{x+\frac{1}{x}} dx$$

$$I = \int e^{x+\frac{1}{x}} dx + \underbrace{\int x\left(1-\frac{1}{x^2}\right) e^{x+\frac{1}{x}} dx}_{\text{Let the function be k and use by parts}} \quad \text{---} \ (1)$$

$$k = \int x\left(1-\frac{1}{x^2}\right) e^{x+\frac{1}{x}} dx$$

$$k = x \int \left(1-\frac{1}{x^2}\right) e^{x+\frac{1}{x}} dx - \int \frac{dx}{dx} \int \left(1-\frac{1}{x^2}\right) e^{x+\frac{1}{x}} dx \cdot dx + c$$

let $x+\frac{1}{x} = t$

$\left(1-\frac{1}{x^2}\right) dx = dt$

then $= \int \left(1-\frac{1}{x^2}\right) e^{x+\frac{1}{x}} dx$

$= \int e^t dt = e^t$

$$\therefore \ k = x e^{x+\frac{1}{x}} - \int e^{x+\frac{1}{x}} dx + c$$

Now,

$$I = \cancel{\int e^{x+\frac{1}{x}} dx} + x e^{x+\frac{1}{x}} - \cancel{\int e^{x+\frac{1}{x}} dx} + c$$

$$I = x e^{x+\frac{1}{x}} + c$$