JEE Advanced Problems

EXAMPLE

Find the sum of all possible values of c for which the expression $x^3 + 3x^2 - 9x + c$ can be expressed as the product of three factors, two of which are identical and monic.

$A) 2$ $B) -22$
$C) -2$ $D) -27$

Concepts tested: Factorization

Answer: B) -22

Solution:
Step 1: Figure out the appropriate factors
Let the identical and monic factor be $x + a$ and then the remaining factor will be $x + \frac{c}{a^2}$. So, we can say that $x^3 + 3x^2 - 9x + c = (x + a)^2(x + \frac{c}{a^2})$.

Step 2: Comparing the coefficients and finding the values of c
On comparing the coefficients, we get two equations : $\frac{c}{a^2} + 2a = 3$ and $\frac{2c}{a} + a^2 = -9$. On finding the value of $\frac{c}{a}$ from first equation i.e. $\frac{c}{a} = 3a - 2a^2$ and substituting in the second one, we get $2(3a - 2a^2) + a^2 = -9 \Rightarrow 3a^2 - 6a - 9 = 0 \Rightarrow a = 3, -1$ and thus the corresponding values of c are $-27, 5$.

Step 3: Summing all the values
Hence the sum of the possible values of c is $-27 + 5 = -22$.

Common mistakes:

- If you didn't compare the coefficients carefully, you will obtain incorrect equations and hence incorrect values of c.