

EXAMPLE

Suppose the inequality $x^2 + y^2 + xy + 1 \geq h(x + y)$ holds for all real x and y , then find the sum of all possible integral values of h .

A) 1
C) ∞

B) -1
D) 0

Concepts tested: Discriminant

Answer: D) 0

Solution:

Step 1: Finding sign of the quadratic equation in terms of x

$$x^2 + x(y - h) + y^2 - hy + 1 \geq 0 \quad \forall x \in \mathbb{R}$$

$$\Rightarrow (y - h)^2 - 4(y^2 - hy + 1) \leq 0 \quad (\text{Discriminant} \leq 0)$$

$$\Rightarrow -3y^2 + 2hy + h^2 - 4 \leq 0$$

$$\therefore 3y^2 - 2hy + 4 - h^2 \geq 0 \quad \forall y \in \mathbb{R}$$

Step 2: Finding the sign of quadratic inequality in terms of y

$$3y^2 - 2hy + 4 - h^2 \geq 0$$

$$\Rightarrow 4h^2 - 4 \cdot 3(4 - h^2) \leq 0 \quad (\text{Discriminant} \leq 0)$$

$$\Rightarrow h^2 - 3(4 - h^2) \leq 0$$

$$\Rightarrow 4h^2 - 12 \leq 0 \Rightarrow h \in [-\sqrt{3}, \sqrt{3}]$$

Step 3: Finding and summing the values of h

The integral values of h lying in the interval $[-\sqrt{3}, \sqrt{3}]$ are $-1, 0, 1$. Hence the sum of all suitable values of h is 0.