JEE Mains Problems

EXAMPLE

For what real values of p, will the equation

$$(p^2 - 16)x^2 - (4 + 5p + p^2)x - p^3 - 4p^2 - 4p - 16 = 0$$

have more than two solutions?

A) No such value of
$$p$$
 exists

$$B)4,-4$$

$$(C) - 4$$

$$D)4,-4,-1$$

Concepts tested: Identity

Answer: C) -4

For the quadratic equation $(p^2-16)x^2-(4+5p+p^2)x-p^3-4p^2-4p-16=0$ to have more than two solutions, it must be an identity. Therefore,

$$p^2 - 16 = 4 + 5p + p^2 = -p^3 - 4p^2 - 4p - 16 = 0.$$

Solving these, we get

$$p^2 - 16 = 0 \implies p = \pm 4 \tag{1}$$

$$4 + 5p + p^2 = 0 \implies p = -1, -4$$
 (2)

$$\begin{array}{cccc}
 p & -10 & -0 & \longrightarrow & p & -14 & & (1) \\
 4 + 5p + p^2 & 0 & \Longrightarrow & p & -1, -4 & & (2) \\
 -p^3 - 4p^2 - 4p - 16 & = 0 & \Longrightarrow & (p+4)(p^2+4) & = 0 & \Longrightarrow & p & = -4. & (3)
 \end{array}$$

Taking the intersection of (1), (2), and (3), we get p=-4.

Common mistakes:

• If you take the union of the values of p which you got after equating the coefficients to zero, you will end up with option D), which is wrong.