EXAMPLE

If $P(x) = ax^2 + bx + c$ and $Q(x) = ax^2 + ax - c$, provided that $ac \neq 0$, then which of the following is true about the roots of the equation $P(x).Q(x) = 0$?

$A)$ four real roots
$B)$ exactly two real roots
$C)$ at most two real roots
$D)$ either two or four real roots

Concepts tested: Discriminant

Answer: D) either two or four real roots

Solution:
$P(x).Q(x) = 0$ implies either $P(x) = 0$ or $Q(x) = 0$. Let $D_1 = b^2 - 4ac$ be the discriminant of the equation $P(x) = 0$ and $D_2 = a^2 + 4ac$ be the discriminant of the equation $Q(x) = 0$. If $4ac > 0$, then $D_2 > 0$, which implies $Q(x) = 0$ has two real and distinct roots; if $4ac < 0$, then $D_1 > 0$, which implies $P(x) = 0$ has two real and distinct roots. Hence, in any case, the equation $P(x).Q(x) = 0$ has either two or four real roots.

Common mistakes:

- If you expanded the product of $P(x)$ and $Q(x)$, you would be left with a 4^{th}-degree equation and we don't know how to easily solve a 4^{th}-degree equation.