EXAMPLE

Let a, b be the roots of the equation $x^2 + 13x - 2 = 0$, then find the quadratic equation whose roots are $\frac{a}{a+10}$ and $\frac{b}{b+10}$.

$A)\ -32x^2 + 134x - 2 = 0$ $\quad$ $B)\ 32x^2 + 134x - 2 = 0$

$C)\ 111x^2 - 9x - 2 = 0$ $\quad$ $D)\ 111x^2 + 9x + 2 = 0$

Concepts tested: Transformation of equation

Answer: A) $-32x^2 + 134x - 2 = 0$

Solution:

Let $y = \frac{x}{x+10}$, where x can take values a, b. Then $xy + 10y = x \implies x = \frac{10y}{1-y}$. On substituting the value of x in $x^2 + 13x - 2 = 0$, we get

$$\left(\frac{10y}{1-y}\right)^2 + 13\left(\frac{10y}{1-y}\right) - 2 = 0$$
$$100y^2 + 13(10y)(1-y) - 2(1-y)^2 = 0$$
$$-32y^2 + 134y - 2 = 0.$$

Writing this in terms of the variable x, we get $-32x^2 + 134x - 2 = 0$.

Common mistakes:

- If you tried to find the new equation by using the exact roots of the given equation, it will lead you to a lot of additional, complicated calculations.