

The positive integer just greater than $(1 + 0.0001)^{10000}$ is

(a) 4 (b) 5 (c) 2 (d) 3

Answer: (d)

Solution:

$(1 + 0.0001)^{10000}$ which is similar to the form $(1 + 1/n)^n$ where $n = 10000$.

Using binomial expansion, we have

$$(1 + 0.0001)^{10000} = (1 + 1/n)^n$$

$$= 1 + n \times 1/n + \frac{n(n-1)}{2!} \times 1/n^2 + \frac{[n(n-1)(n-2)]}{3!} \times 1/n^3 + \dots$$

$$= 1 + 1 + \frac{1}{2!}(1 - 1/n) + \frac{1}{3!}(1 - 1/n) + (1 - 2/n) + \dots$$

$$< 1 + 1/1! + 1/2! + 1/3! + \dots + 1/(9999)!$$

$$= 1 + 1/1! + 1/2! + 1/3! + \dots \infty$$

$$= e < 3$$