Find the term independent of x, $x \neq 0$ in the expansion of $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^{15}$

Sol. General Term $T_{r+1} = {}^nC_r x^{n-r} y^r$

$$= {}^{15}C_r \left(\frac{3x^2}{2}\right)^{15-r} \left(-\frac{1}{3x}\right)^r = {}^{15}C_r \left(\frac{3}{2}\right)^{15-r} \cdot (x)^{30-2r} \cdot \left(-\frac{1}{3}\right)^r \cdot \frac{1}{x^r}$$

$$= {}^{15}C_r \left(\frac{3}{2}\right)^{15-r} \cdot (x)^{30-2r-r} (-1)^r \cdot \frac{1}{(3)^r}$$

$$= {}^{15}C_r \left(\frac{3}{2}\right)^{15-r} \cdot x^{30-3r} (-1)^r \cdot \frac{1}{(3)^r}$$

for getting the term independent of x,

$$30 - 3r = 0 \quad \Rightarrow \quad r = 10$$

On putting the value of r in the above expression, we get

$$= {}^{15}C_{10} \left(\frac{3}{2}\right)^{15-10} (-1)^{10} \cdot \frac{1}{(3)^{10}} = {}^{15}C_{10} \frac{(3)^5}{(2)^5} \cdot \frac{1}{(3)^{10}}$$

$$= {}^{15}C_{10} \cdot \frac{1}{(2)^5 \cdot (3)^5} = {}^{15}C_{10} \left(\frac{1}{6}\right)^5$$

Hence, the required term $= {}^{15}C_{10} \left(\frac{1}{6}\right)^5$