For example, let $A = \begin{bmatrix} 3 & 1 \\ -5 & x \end{bmatrix}$, then – A is given by

$$-A = (-1)A = (-1)\begin{bmatrix} 3 & 1 \\ -5 & x \end{bmatrix} = \begin{bmatrix} -3 & -1 \\ 5 & -x \end{bmatrix}$$

Difference of matrices If A = $[a_{ij}]$, B = $[b_{ij}]$ are two matrices of the same order, say $m \times n$, then difference A – B is defined as a matrix D = $[d_{ij}]$, where $d_{ij} = a_{ij} - b_{ij}$, for all value of i and j. In other words, D = A – B = A + (–1) B, that is sum of the matrix A and the matrix – B.

Example 7 If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$, then find 2A – B.

Solution We have

$$2A - B = 2\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \end{bmatrix} + \begin{bmatrix} -3 & 1 & -3 \\ 1 & 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-3 & 4+1 & 6-3 \\ 4+1 & 6+0 & 2-2 \end{bmatrix} = \begin{bmatrix} -1 & 5 & 3 \\ 5 & 6 & 0 \end{bmatrix}$$

3.4.3 *Properties of matrix addition*

The addition of matrices satisfy the following properties:

(i) **Commutative Law** If A = $[a_{ij}]$, B = $[b_{ij}]$ are matrices of the same order, say $m \times n$, then A + B = B + A.

Now A + B = $[a_{ij}] + [b_{ij}] = [a_{ij} + b_{ij}]$
$= [b_{ij} + a_{ij}]$ (addition of numbers is commutative)
$= ([b_{ij}] + [a_{ij}])$ = B + A

(ii) **Associative Law** For any three matrices A = $[a_{ij}]$, B = $[b_{ij}]$, C = $[c_{ij}]$ of the same order, say $m \times n$, (A + B) + C = A + (B + C).

Now (A + B) + C = $([a_{ij}] + [b_{ij}]) + [c_{ij}]$
$= [a_{ij} + b_{ij}] + [c_{ij}] = [(a_{ij} + b_{ij}) + c_{ij}]$
$= [a_{ij} + (b_{ij} + c_{ij})]$ (Why?)
$= [a_{ij}] + [(b_{ij} + c_{ij})] = [a_{ij}] + ([b_{ij}] + [c_{ij}])$ = A + (B + C)

(iii) **Existence of additive identity** Let A = [a_{ij}] be an $m \times n$ matrix and O be an $m \times n$ zero matrix, then A + O = O + A = A. In other words, O is the additive identity for matrix addition.

(iv) **The existence of additive inverse** Let A = $[a_{ij}]_{m \times n}$ be any matrix, then we have another matrix as – A = $[-a_{ij}]_{m \times n}$ such that A + (– A) = (– A) + A= O. So – A is the additive inverse of A or negative of A.

3.4.4 *Properties of scalar multiplication of a matrix*

If A = [a_{ij}] and B = [b_{ij}] be two matrices of the same order, say $m \times n$, and k and l are scalars, then

(i) k(A +B) = k A + kB, (ii) (k + l)A = k A + l A

(ii) $k\,(A + B) = k\,([a_{ij}] + [b_{ij}])$

$$= k\,[a_{ij} + b_{ij}] = [k\,(a_{ij} + b_{ij})] = [(k\,a_{ij}) + (k\,b_{ij})]$$

$$= [k\,a_{ij}] + [k\,b_{ij}] = k\,[a_{ij}] + k\,[b_{ij}] = kA + kB$$

(iii) $(k + l)\,A = (k + l)\,[a_{ij}]$

$$= [(k + l)\,a_{ij}] + [k\,a_{ij}] + [l\,a_{ij}] = k\,[a_{ij}] + l\,[a_{ij}] = k\,A + l\,A$$

Example 8 If $A = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$, then find the matrix X, such that 2A + 3X = 5B.

Solution We have 2A + 3X = 5B

or $\quad 2A + 3X - 2A = 5B - 2A$

or $\quad 2A - 2A + 3X = 5B - 2A$ (Matrix addition is commutative)

or $\quad O + 3X = 5B - 2A$ (– 2A is the additive inverse of 2A)

or $\quad 3X = 5B - 2A$ (O is the additive identity)

or $$X = \frac{1}{3}(5B - 2A)$$

or $$X = \frac{1}{3}\left(5\begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix} - 2\begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}\right) = \frac{1}{3}\left(\begin{bmatrix} 10 & -10 \\ 20 & 10 \\ -25 & 5 \end{bmatrix} + \begin{bmatrix} -16 & 0 \\ -8 & 4 \\ -6 & -12 \end{bmatrix}\right)$$