

27. Find the equation of a circle passing through the point (7, 3) having radius 3 units and whose centre lies on the line $y = x - 1$.

Solution:

Since, the equation of a circle having centre (h, k), having radius as r units, is

$$(x - h)^2 + (y - k)^2 = r^2 \text{ Centre lies on the line i.e., } y = x - 1,$$

Co - Ordinates are (h, k) = (h, h - 1)

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(7 - h)^2 + (3 - (h - 1))^2 = 3^2$$

$$49 + h^2 - 14h + (3 - h + 1)^2 = 9$$

On rearranging we get

$$h^2 - 14h + 49 + 16 + h^2 - 8h - 9 = 0$$

$$2h^2 - 22h + 56 = 0$$

$$h^2 - 11h + 28 = 0$$

$$h^2 - 4h - 7h + 28 = 0$$

$$h(h - 4) - 7(h - 4) = 0$$

$$(h - 7)(h - 4) = 0$$

$$h = 7 \text{ or } 4$$

Centre = (7, 6) or (4, 3)

$$(x - h)^2 + (y - k)^2 = r^2$$

Equation, having centre (7, 6)

$$(x - 7)^2 + (y - 6)^2 = 3^2$$

$$x^2 - 14x + 49 + y^2 - 12y + 36 - 9 = 0$$

$$x^2 - 14x + y^2 - 12y + 76 = 0$$

Equation, having centre (4, 3)

$$(x - 4)^2 + (y - 3)^2 = 3^2$$

$$x^2 - 8x + 16 + y^2 - 6y + 9 - 9 = 0$$

$$x^2 - 8x + y^2 - 6y + 16 = 0$$