50. Let $f(x)$ be a polynomial of degree 4 having extreme values at $x = 1$ and $x = 2$. If $\lim_{x \to 0}\left(\frac{f(x)}{x^2}+1\right) = 3$ then $f(-1)$ is equal to **[Online April 15, 2018]**

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{9}{2}$

(d) $\because f(x)$ has extremum values at $x = 1$ and $x = 2$

$\because f'(1) = 0$ and $f'(2) = 0$

As, $f(x)$ is a polynomial of degree 4.

Suppose $f(x) = Ax^4 + Bx^3 + Cx^2 + Dx + E$

$$\because \quad \lim_{x \to 0}\left(\frac{f(x)}{x^2}+1\right) = 3$$

$$\Rightarrow \quad \lim_{x \to 0}\left(\frac{Ax^4 + Bx^3 + Cx^2 + Dx + E}{x^2}+1\right) = 3$$

$$\Rightarrow \quad \lim_{x \to 0}\left(Ax^2 + Bx + C + \frac{D}{x} + \frac{E}{x^2}+1\right) = 3$$

As limit has finite value, so $D = 0$ and $E = 0$

Now $A(0)^2 + B(0) + C + 0 + 0 + 1 = 3$

$\Rightarrow c + 1 = 3 \Rightarrow c = 2$

$f'(x) = 4Ax^3 + 3Bx^2 + 2Cx + D$

$f'(1) = 0 \Rightarrow \quad 4A(1) + 3B(1) + 2C(1) + D = 0$

$\Rightarrow \quad 4A + 3B = -4 \quad$ ...(i)

$f'(2) = 0 \Rightarrow \quad 4A(8) + 3B(4) + 2C(2) + D = 0$

$\Rightarrow \quad 8A + 3B = -2 \quad$ ...(ii)

From equations (i) and (ii), we get

$A = \frac{1}{2}$ and $B = -2$

So, $f(x) = \frac{x^4}{2} - 2x^3 + 2x^2$

Therefore, $f(-1) = \frac{(-1)^4}{2} - 2(-1)^3 + 2(-1)^2$

$= \frac{1}{2} + 2 + 2 = \frac{9}{2}$. Hence $f(-1) = \frac{9}{2}$