

4. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \frac{x}{1+x^2}$, $x \in \mathbb{R}$. Then the range of f is : [Jan. 11, 2019 (I)]

- (a) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (b) $\mathbb{R} - [-1, 1]$
(c) $\mathbb{R} - \left[-\frac{1}{2}, \frac{1}{2}\right]$ (d) $(-1, 1) - \{0\}$

4. (a) $f(x) = \frac{x}{1+x^2}, x \in \mathbb{R}$

$$\text{Let, } y = \frac{x}{1+x^2}$$

$$\Rightarrow yx^2 - x + y = 0 \Rightarrow x = \frac{1 \pm \sqrt{1-4y^2}}{2}$$

$$\Rightarrow 1 - 4y^2 \geq 0$$

$$\Rightarrow 1 \geq 4y^2$$

$$\Rightarrow |y| \leq \frac{1}{2}$$

$$\Rightarrow -\frac{1}{2} \leq y \leq \frac{1}{2}$$

$$\Rightarrow \text{The range of } f \text{ is } \left[-\frac{1}{2}, \frac{1}{2}\right].$$

