1. Let R_1 and R_2 be two relations defined as follows :

$R_1 = \{(a, b) \in \mathbf{R}^2 : a^2 + b^2 \in Q\}$ and

$R_2 = \{(a, b) \in \mathbf{R}^2 : a^2 + b^2 \notin Q\}$, where Q is the set of all rational numbers. Then : **[Sep. 03, 2020 (II)]**

(a) Neither R_1 nor R_2 is transitive.
(b) R_2 is transitive but R_1 is not transitive.
(c) R_1 is transitive but R_2 is not transitive.
(d) R_1 and R_2 are both transitive.

1. (a) For R_1 let $a = 1+\sqrt{2}$, $b = 1-\sqrt{2}$, $c = 8^{1/4}$

$aR_1b \Rightarrow a^2 + b^2 = (1+\sqrt{2})^2 + (1-\sqrt{2})^2 = 6 \in Q$

$bR_1c \Rightarrow b^2 + c^2 = (1-\sqrt{2})^2 + (8^{1/4})^2 = 3 \in Q$

$aR_1c \Rightarrow a^2 + c^2 = (1+\sqrt{2})^2 + (8^{1/4})^2 = 3 + 4\sqrt{2} \notin Q$

$\therefore R_1$ is not transitive.

For R_2 let $a = 1+\sqrt{2}$, $b = \sqrt{2}$, $c = 1-\sqrt{2}$

$aR_2b \Rightarrow a^2 + b^2 = (1+\sqrt{2})^2 + (\sqrt{2})^2 = 5 + 2\sqrt{2} \notin Q$

$bR_2c \Rightarrow b^2 + c^2 = (\sqrt{2})^2 + (1-\sqrt{2})^2 = 5 - 2\sqrt{2} \notin Q$

$aR_2c \Rightarrow a^2 + c^2 = (1+\sqrt{2})^2 + (1-\sqrt{2})^2 = 6 \in Q$

$\therefore R_2$ is not transitive.