

Find $\sum_{r=1}^n \frac{1 + 2 + 3 + \dots + r}{r}$

SOLUTION

$$\begin{aligned} & \sum_{r=1}^n \frac{1 + 2 + 3 + \dots + r}{r} \\ &= \sum_{r=1}^n \frac{r(r+1)}{2r} \\ &= \frac{1}{2} \sum_{r=1}^n (r+1) \\ &= \frac{1}{2} \left[\sum_{r=1}^n r + \sum_{r=1}^n 1 \right] \\ &= \frac{1}{2} \left[\frac{n(n+1)}{2} + n \right] \\ &= \frac{n}{4} [(n+1) + 2] \\ &= \frac{n}{4} (n+3). \end{aligned}$$