Find $\sum_{r=1}^{n}(3r^2 - 2r + 1)$.

SOLUTION

$$\sum_{r=1}^{n}(3r^2 - 2r + 1)$$

$$= 3\sum_{r=1}^{n} r^2 - 2\sum_{r=1}^{n} r + \sum_{r=1}^{n} 1$$

$$= 3.\frac{n(n+1)(2n+1)}{6} - 2\frac{n(n+1)}{2} + n$$

$$= \frac{n}{2}\left[2n^2 + 3n + 1\right) - 2(n+1) + 2\Big]$$

$$= \frac{n}{2}\left(2n^2 + 3n + 1 - 2n - 2 + 2\right)$$

$$= \frac{n}{2}\left(2n^2 + n + 1\right).$$