Q. **In many experimental set-ups, the source and screen are fixed at a distance say *D* and the lens is movable. Show that there are two positions for the lens for which an image is formed on the screen. Find the distance between these points and the ratio of the image sizes for these two points.**

K **Thinking Process**

This is also one of the methods for finding the focal length of the lens in laboratory and known as displacement method.

Ans. Principal of reversibility is states that the position of object and image are interchangeable So, by the versibility of u and v, as seen from the formula for lens.

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

It is clear that there are two positions for which there shall be an image.

On the screen, let the first position be when the lens is at O. Finding u and v and substituting in lens formula

Given, $$-u + v = D$$

$\Rightarrow$ $$u = -(D - v)$$

Placing it in the lens formula

$$\frac{1}{D - v} + \frac{1}{v} = \frac{1}{f}$$

On solving, we have

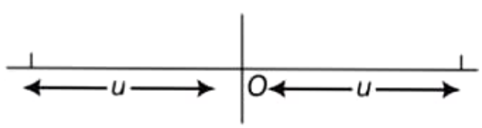
$\Rightarrow$ $$\frac{v + D - v}{(D - v)v} = \frac{1}{f}$$

$\Rightarrow$ $$v^2 - Dv + Df = 0$$

$\Rightarrow$ $$v = \frac{D}{2} \pm \frac{\sqrt{D^2 - 4Df}}{2}$$

Hence, finding u

$$u = -(D - v) = -\left(\frac{D}{2} \pm \frac{\sqrt{D^2 - 4Df}}{2}\right)$$



When, the object distance is $\frac{D}{2} + \frac{\sqrt{D^2 - 4Df}}{2}$

the image forms at $\frac{D}{2} - \frac{\sqrt{D^2 - 4Df}}{2}$

Similarly, when the object distance is

$$\frac{D}{2} - \frac{\sqrt{D^2 - 4Df}}{2}$$

The image forms at $\frac{D}{2} + \frac{\sqrt{D^2 - 4Df}}{2}$

The distance between the poles for these two object distance is

$$\frac{D}{2} + \frac{\sqrt{D^2 - 4Df}}{2} - \left(\frac{D}{2} - \frac{\sqrt{D^2 - 4Df}}{2}\right) = \sqrt{D^2 - 4Df}$$

Let $$d = \sqrt{D^2 - 4Df}$$

If $u = \frac{D}{2} + \frac{d}{2}$, then the image is at $v = \frac{D}{2} - \frac{d}{2}$.

$\therefore$ The magnification $$m_1 = \frac{D - d}{D + d}$$

If $u = \frac{D - d}{2}$, then $v = \frac{D + d}{2}$

$\therefore$ The magnification $$m_2 = \frac{D + d}{D - d}$$

Thus, $$\frac{m_2}{m_1} = \left(\frac{D + d}{D - d}\right)^2.$$

This is the required expression of magnification.